

Branes and Fluxes in Orientifolds and K-theory

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Abstract

RR fields in string backgrounds including orientifold planes and branes on top of them are classified by K-theory. Following the idea introduced in hep-th/0103183, we also classify such fluxes by cohomology. Both of them are compared through the Atiyah-Hirzebruch Spectral Sequence. Some new correlations between branes on orientifold planes Op^\pm and obstructions to the existence of some branes are found. Finally, we find a topological condition that avoid the presence of global gauge anomalies in lower dimensional systems.

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1 Introduction

String theory backgrounds including orientifold planes have been studied in detail (see [1, 2, 3, 4, 5, 6, 7, 8, 9]). Many features of supersymmetric and non-supersymmetric gauge theories have been understood since the introduction of orientifolds as perturbative and nonperturbative string backgrounds. One of them is the existence of many types of orientifolds, which arise when discrete fluxes are turned on. The existence of these fluxes associated to NS-NS and R-R sectors of the theory, change the charge and tension of orientifold planes. For instance, some of them carry half-integer values of RR charge violating the Dirac quantization condition. In other cases, planes with the same dimensionality have a relative charge differing by one half from each other, in D-brane units of charge.

There are at least two different (but related) ways to turn on such discrete fluxes. One of them uses the fact that branes can end on branes giving a “brane realization of discrete torsion” (for more details see section two). The second one is the classification of orientifolds provided by cohomology. In fact, cohomology groups of the transversal space to orientifolds, classify RR fluxes in the bulk space. It turns out that some of these fluxes are actually discrete torsion, which in turn, describe the existence of a new type of orientifold plane. However, cohomology, in general *do not* provides a satisfactory explanation of why some of these orientifolds have a fractional relative charge and moreover, why some of them (actually those with a spatial dimension less than 5) have indeed a fractional RR charge.

The problem of the relative charge among some orientifold planes, is successfully resolved by K-theory (although the problem of the fractional charge for a single orientifold plane is still open). K-theory has been proved to be a very fruitful mathematical tool to classify D-branes in string theory (see [10, 11, 12]). Originally, K-theory was used to classify RR charges in different backgrounds [13, 14, 15, 16, 17]. Recently it was proved that K-theory also classifies RR fields [18], i.e., the fields related to the D-branes at points far away from orientifold planes (for a formal treatment, see [19]). Using this result in Ref. [5], it was possible to classify RR fields in the presence of orientifold backgrounds as well (with no extra D-branes). Since some of these fluxes turn out to be discrete torsion in the presence of orientifold planes, such a classification is also an orientifold classification, this time provided by K-theory. In fact, comparing the cohomology and K-theory results, it was possible to explain the relative RR charge among some orientifolds and moreover, new features were found, such as the absence of certain orientifolds as well as the equivalence between other ones (i.e., some orientifold

planes seemed to be different in the former cohomology classification but they turn out to be the same object in the K-theory perspective).

In this paper, we are interested in classifying RR fields by K-theory in string theory backgrounds including orientifold planes and d -branes¹. In particular, we consider the case of d -branes on top of the orientifold planes, i.e., only those oriented parallel to the orientifolds. The orientifold planes and some of these branes can be regarded as the T-dual versions of the D-branes in Type I and Type $USp(32)$ [20] string theories (which have an $O9^-$ and an $O9^+$ -planes respectively) when T-duality is taken over their longitudinal coordinates (if the number of compact coordinates is higher than the dimensionality of the D-brane, its T-dual version will be a D-brane transversal to the orientifold plane; such branes are not considered in the present paper). Also we find new correlations between RR fields in the presence of the branes on top of orientifolds. In order to do this, we require the knowledge of a cohomology classification. This give us an alternative method to classify orientifolds by cohomology, when the dimension of the brane is equal to that of the orientifold. The method consist in wrapping $D(d+n)$ -branes on n -cycles of homology to get Dd -branes. There are certain restrictions in which branes can be wrapped as well as which cycles are considered, but once we fixed the homology cycles, we are able to compute the corresponding homology group which classify them. By Poincaré duality we get the required cohomology group. It is important to point out that our results are in agreement with the above mentioned cohomology classification of orientifolds.

By comparing both results (K-theory and cohomology) we obtain some new correlations among RR fields and branes. Such a comparison is made by using the Atiyah-Hirzebruch Spectral Sequence (AHSS). Among other important results, we find that K-theory fixes the topological conditions to cancel global anomalies arising in probe branes within the same backgrounds we are considering in this paper.

The paper is organized as follows: in section 2 we briefly survey some important aspects of orientifolds. In section 3 we review how to calculate RR charges of branes on top of an Op^\pm -plane. Here we describe the T-dual version in which we restrict our study throughout this paper. Also we discuss on discrete charge cancellation on compact spaces that are reflected on global gauge anomaly cancellation on suitable probe branes. The $\widehat{D3}$ -brane in Type $USp(32)$ string theory is discussed.

In section 4 we begin by reviewing the classification of RR fields through K-theory

¹Throughout this paper, we are using the following notation: d -branes stands for a d -dimensional brane on top of an Op -plane for which we do not know neither its charge nor its nature (i.e., if it is a Dirichlet brane or another type of brane).

and describing how the AHSS works by relating cohomology to K-theory. After that we give the relevant K-theory group which classifies RR fields in orientifolds and d -branes backgrounds.

In section 5, we show how to obtain possible d -branes on top of orientifold p -planes by wrapping $D(d+n)$ -branes on non-trivial and compact n -cycles on the projective spaces \mathbb{RP}^{8-p} .

In section 6 we apply the AHSS to relate the results given by K-theory classification of RR fields, and those given by cohomology. We interpret the results in the spirit of Ref. [5]. This is done for all $d < p$ and for $0 < p \leq 6$. Finally we give our conclusions in section 7. Also in the appendix A we give detail aspects of transforming fluxes into branes. Some important remarks about T-duality on such branes are considered in appendix B.

2 Overview on Orientifolds

In this section we review some important aspects about orientifold planes (see for instance [2, 3, 4, 5, 6, 8]). Our aim is not to provide an extensive review of orientifolds but to briefly recall some of their relevant properties.

An orientifold plane in Type II superstring theory is defined as the plane conformed by the loci of fixed points under the action of a discrete symmetry \mathcal{I}_{9-p} , which reverses the transverse $(9-p)$ coordinates, and that of Ω which reverses the string worldsheet orientation. Hence an orientifold Op is given by the plane

$$\mathbb{R}^{p+1} \times \left(\mathbb{R}^{9-p} / \Omega \cdot \mathcal{I}_{9-p} \cdot J \right), \quad (2.1)$$

with J given by (see [21])

$$J = \begin{cases} 1 & p = 0, 1 \bmod 4 \\ (-1)^{F_L} & p = 2, 3 \bmod 4 \end{cases}. \quad (2.2)$$

There are at least two different types of orientifold planes, denoted as Op^\pm , where \pm stands for the sign of the RR charge they carry on. Actually, they carry a RR charge equal to $\pm 2^{p-5}$ in D-brane charge units (notice that for $p < 5$, the orientifold plane has a fractional charge). These two different types of orientifold planes can be regarded as arising (via T-duality) from the nine-dimensional orientifold planes $O9^\pm$, which in turn, establish the existence of the ten-dimensional string theories, known as type I ($O9^-$) and type $USp(32)$ ($O9^+$) theories. In the former one, 16 D9-branes (and their images) are needed in order to cancel the -16 charge (in D-brane charge units) due to the orientifold $O9^-$. The physical states (coming from the quantized open string)

are those which survive the action of the projection operator $\widehat{P}_\Omega = \frac{1}{2}(1 + \widehat{\Omega})$, i.e., the surviving states have an eigenvalue equal to one under the $\widehat{\Omega}$ action, which acts on the Chan-Paton factors as

$$\widehat{\Omega}\lambda|\Psi\rangle = \gamma_\Omega^{-1}\lambda^T\gamma_\Omega|\Omega\Psi\rangle, \quad (2.3)$$

with $\lambda \in SO(n)$ and γ_Ω being the generating element of a representation of the \mathbf{Z}_2 acting on the Chan-Paton labels, satisfying $\gamma_\Omega = -\gamma_\Omega^T$. On the other hand, for states satisfying $\gamma_\Omega = \gamma_\Omega^T$ ($\widehat{\Omega}^2 = 1$) the gauge group is $USp(n)$ and the RR charge of the orientifold plane is positive (+16). By taking this orientifold plane, it is possible to construct the so called $USp(32)$ string theory², which was proposed in [20]. The above can be summarized as follows:

$$\begin{aligned} \text{Type I} & : \quad 32 \text{ D9} + O9^- + \text{IIB} \\ \text{Type USp(32)} & : \quad 32 \overline{\text{D9}} + O9^+ + \text{IIB}. \end{aligned}$$

By taking T-duality on $(9-p)$ spatial coordinates on the orientifolds $O9^\pm$ (i.e. in Type I and $USp(32)$ theories), we get the Op^\pm orientifolds (actually $2^{(9-p)}$ of them) and also T-dual versions of D-branes in the above two ten-dimensional string theories. We focus our attention in Dd-branes on top of Op^\pm planes (i.e., D-branes with only longitudinal coordinates with respect to the orientifold plane). They come from $D(d+9-p)$ -branes in the ten-dimensional theories which are wrapped on the compact $(9-p)$ -coordinates. Also, as we know from K-theory, the D-branes present in both ten-dimensional string theories are D9, D5, D1 (BPS states) and $\widehat{\text{D8}}$ and $\widehat{\text{D7}}$ (non-BPS states) for Type I string theory, while for Type $USp(32)$ the difference lies on the non-BPS spectrum of branes, which in this case is given by $\widehat{\text{D4}}$ and $\widehat{\text{D3}}$ branes.

On the other hand, an orientifold classification can also be provided by a non-perturbative analysis. This classification is given by cohomology and by turning on discrete fluxes. Before of reviewing this classification let us start by the analysis of discrete NS fluxes. The transverse space to Op (actually the projective space \mathbb{RP}^{8-p}) contains a set of non-trivial homology cycles where D-branes or NS5-branes can be

²In the usual context, the symplectic group appears by imposing the conditions $\widehat{\Omega}^2 = 1$, physical states with eigenvalue $\Omega = 1$ and $\gamma_\Omega = -\gamma_\Omega^T$. However, alternatively we can impose $\widehat{\Omega}^2 = 1$, physical states with eigenvalue $\Omega = -1$ and $\gamma_\Omega = \gamma_\Omega^T$, obtaining also the symplectic gauge group. Tadpole cancellation condition fixes the range of the gauge group to be 32. This give rise to the $USp(32)$ string theory with one $O9^+$ -plane. Since the $U(1)$ gauge boson, present in the spectrum of a single D-brane and the NS B -field, are both odd under the orientifold projection, the difference between Type I and $USp(32)$ string theories lies in the fact that in the former one the fields are projected out while in the latter one they are not. The two orientifolds $O9^\pm$ differ from each other by the presence of a non-trivial NS-NS two-form.

wrapped on. Actually, this picture is “the brane realization of discrete torsion”, where an Op^+ -plane can be constructed by the intersection of an Op^- and a NS5-brane. Hence, it is important to study the action of an Op^- -plane on the B -field (for which the NS5-brane is the magnetic source). It turns out that B is odd under the orientifold projection, which means that $H = dB$ is classified by a torsion cohomology group³. Hence, $[H_{NS} = dB_{NS}] \in H^3(\mathbb{RP}^{8-p}, \tilde{\mathbb{Z}}) = \mathbf{Z}_2$, with $\tilde{\mathbb{Z}}$ being the twisted sheaf-bundle of integers [4]. The trivial class of the two-torsion discrete group stands for the presence of an Op^- while the non-trivial one is related to the Op^+ . This can be understood as follows.

The B -field has a non-trivial holonomy given by

$$b = \oint_{\mathbb{RP}^2} \frac{B}{2\pi} = \frac{1}{2} \quad (2.4)$$

with $\mathbb{RP}^2 \subset \mathbb{RP}^{8-p}$ surrounding the Op -plane. This holonomy contributes by a factor

$$g = e^{i \int_{\mathbb{RP}^2} B} = e^{i\pi} = -1 \quad (2.5)$$

to the Möbius strip amplitude \mathcal{M}_2 . Suppose we start with an Op^- -plane, hence

$$\mathcal{M}_2 \sim Tr \frac{1}{4} \Omega g (1 + (-1)^F) e^{-Ht}. \quad (2.6)$$

Then instead of having states invariant under $\frac{1}{2}(1 + \hat{\Omega})$ they are invariant under $\frac{1}{2}(1 - \hat{\Omega})$. This means that we have a positive Op^+ -plane (According to the footnote in the page 4). So, the presence of a discrete torsion B -field produces the interchange: $Op^- \longleftrightarrow Op^+$.

Let us turn our attention to the discrete RR fluxes. Orientifold planes establish an action on the RR p' -forms in spacetime. It is important to know how this action affects the fields. The B_{NS} -field in Type I or $USp(32)$ theory, changes its sign under the action of $O9^\pm$ and it remains valid for Op^\pm with other values of p ⁴. However for RR p' -forms, the action depends on the dimension of the orientifold, i.e.,

$$\begin{aligned} \text{untwisted} : C_{p'} &\rightarrow C_{p'} & p' &= p + 1 \text{ mod } 4 \\ \text{twisted} : C_{p'} &\rightarrow -C_{p'} & p' &= p + 3 \text{ mod } 4. \end{aligned} \quad (2.7)$$

There are other kind of orientifolds [3, 4, 5, 6] given by cohomology torsion variants. Forms of an appropriate rank are topologically classified by torsion cohomologies, i.e. $[G_{p'+1}] \in H^{p'+1}(\mathbb{RP}^{8-p}, \mathbb{Z})$, where $G_{p'+1} = dC_{p'}$ (field strength $(p' + 1)$ -form). Twisted

³Roughly speaking, a torsion cohomology group, classifies sections of the bundle $\Omega^3 \otimes \mathcal{E}$ where \mathcal{E} is the non-oriented line bundle over \mathbb{RP}^{8-p} , and Ω^3 is the group of three-forms.

⁴Throughout this paper p stands for the dimensionality of the orientifold plane.

forms given by (2.7) are classified by twisted cohomologies: $H^{p'+1}(\mathbb{RP}^{8-p}, \widetilde{\mathbb{Z}})$. For $p \leq 6$ there are torsion RR fields which are given by

$$[G_{6-p}] \in H^{6-p}(\mathbb{RP}^{8-p}, \mathbb{Z}(\text{or } \widetilde{\mathbb{Z}})) = \mathbf{Z}_2. \quad (2.8)$$

These are background RR discrete fields and they change some properties of orientifold planes.

The main point to focus here is that also (at the cohomological level) RR fields have torsion, as shown in Eq. (2.8). Again there is a non-trivial holonomy factor (for $p \leq 5$ and besides the trivial one) given by

$$c = \oint_{\mathbb{RP}^{5-p}} \frac{C_{5-p}}{2\pi} = \frac{1}{2}, \quad (2.9)$$

that give rise to other kind of orientifold plane denoted by \widetilde{Op} . So we have four types of Op -planes, according to the holonomies (b, c) . The $(0, 0)$ -holonomy represents an Op^- -plane. $(0, 1)$ holonomy is an \widetilde{Op}^- -plane, $(1, 0)$ is an Op^+ -plane and finally a $(1, 1)$ is an \widetilde{Op}^+ -plane.

Gauge groups are $USp(2n)$ for the Op^+ - and \widetilde{Op}^+ -planes, although they differ by their dyon spectrum [4]. For Op^- -plane the gauge group is $SO(2n)$ and for \widetilde{Op}^- -plane is $SO(2n+1)$. By gauge theories and dualities (like the Olive-Montonen duality [3, 4, 5]) it is known that an \widetilde{Op}^- -plane can be thought as the configuration $Op^- + \frac{1}{2}Dp$, where $\frac{1}{2}Dp$ is a fractional (stuck) Dp -brane.

There are extra variants orientifolds \widehat{Op} given by fluxes characterized by cohomology groups H^{2-p} and they are valid only for $p < 2$.

However there are more restrictions. For example, it was shown in Ref. [6] that \widetilde{Op} -planes do not exist for $p \geq 6$, with the exception of the $\widetilde{O6}^-$ -plane, which can be realized as an $O6^-$ immersed in a non-zero background cosmological constant (massive Type IIA supergravity; see section 6.2). Also, we have learned from [5] (see section 4 and 5 for details) that Op^+ and \widetilde{Op}^+ , for $p \leq 3$, are equivalent in K-theory, and moreover, \widehat{Op}^+ and $\widetilde{\widehat{Op}}^+$, do not exist for $p < 2$.

As it was said, turning on discrete fluxes, they can be studied as brane realizations. The b holonomy factor is obtained by intersecting $NS5$ -branes and Op -planes, while the c holonomy factor is obtained by intersecting $D(p+2)$ -branes and Op -planes. Readers interested in the details of these issues are invited to consult Ref. [8] (see also [4, 5]).

3 Dd-branes on Top of Op^\pm -planes

D-branes in type I theory are classified by *real* K-theory⁵ [16, 15, 10, 17], while those in type $USp(32)$, are classified by quaternionic K-theory (see below). If we apply T-duality on $(9 - p)$ compact directions in the above theories, we get a spectrum of D-branes, which are parallel or transversal, to Op -planes. It turns out that real and quaternionic K-theory still classifies such D-branes in the presence of lower dimensional orientifolds. In this section we briefly review how to classify Dd-branes on top of Op -planes, specifically, the ones we are interested in this paper, which are parallelly oriented to the orientifold plane (which in turn means that $d \leq p$). In the second part of this section, we study global gauge anomalies, arising on suitable probe branes on compact spaces, due to the presence of both kinds of orientifold planes, Op^\pm . Our interest in these anomalies, lies in the fact that we will be able to predict the suitable conditions to cancel them, by using K-theory (see section 5).

3.1 K-theory classification of Dd-branes on orientifolds Op^\pm

Before of describing how real K-theory⁶ is used to compute RR charges in string theory (with no B -field in the background) it is useful to give the main properties of these K-theory groups. Consider the following definition $\mathbb{R}^{p,q} := (\mathbb{R}^p/\mathbf{Z}_2) \times \mathbb{R}^q$ where \mathbf{Z}_2 inverts p coordinates. $\mathbb{S}^{p,q}$ is defined as the unitary sphere in $\mathbb{R}^{p,q}$ with dimension $p + q - 1$. Then, $\mathbb{S}^{p,0} \cong \mathbb{RP}^{p-1}$ and $\mathbb{S}^{0,q} \cong \mathbf{S}^{q-1}$.

Real K-theory groups satisfy the following properties:

$$\begin{aligned} KR^{-n}(X) &= KR^{0,n}(X), \\ KR^{p,q}(X) &= KR(X \times \mathbb{R}^{p,q}), \\ KR^{p,q}(X) &= KR^{p+1,q+1}(X) = KR^{p-q}(X), \\ KR^{-m}(X) &= KR^{-m-8}(X). \end{aligned}$$

The same relations are valid for the quaternionic case with $KR^{-n}(X) \cong KH^{-n+4}(X)$. In order to give a complete classification of RR charges on orientifold backgrounds by K-theory, let us describe briefly some results given in Refs. [5, 10, 16, 22].

⁵The action of the worldsheet parity Ω induces an antilinear involution on the gauge bundles E over X that commutes with τ , where τ is the involution $\tau : X \rightarrow X$, $\tau^2 = id$, related to the inversion of transverse coordinates to Op . The Grothendieck group of the isomorphism classes of these bundles is called the *real* K-theory, and it is denoted by $KR(X)$.

⁶For mathematical properties of real K-theory see [15] and for physical applications see Refs. [17, 22, 23, 10].

It was proposed in [10] that real K-theory classifies RR charges of Dd -branes on top of Op^- -planes, while quaternionic K-theory make the same for branes on Op^+ -planes. In [16] this computation was done explicitly.

Also, it was shown in [22] that the K-theory group classifying RR charges in type I T-dual models⁷ is the relative group

$$KR^{p-9}(\mathbf{S}^{p-d} \times \mathbf{T}^{9-p}, \mathbf{T}^{9-p}) \cong \bigoplus_{s=0}^{9-p} \binom{9-p}{s} KO^{-s}(\mathbf{S}^{p-d}), \quad (3.1)$$

with $(9-p)$ being the number of coordinates under which T-duality acts. A similar formula holds for T-dual models of $USp(32)$ string theory, with quaternionic groups.

The groups on the right hand side of (3.1) with $s \neq 9-p$ classify charges for wrapped $D(d+9-p)$ -branes in the ten dimensional theory (with s being the number of wrapped coordinates) and those with $s = 9-p$ classify unwrapped Dd -branes in ten dimensions on an Op -plane.

However, we are interested just in Dd -branes obtained by wrapping $D(d+9-p)$ -branes (either in Type I or $USp(32)$ string theory) on coordinates $d+1, \dots, 9-p$ (notice that we are not interested in branes with transversal coordinates to Op). The real K-theory group classifying these kind of fluxes is given by,

$$KR^{p-9}(\mathbb{R}^{9-p,p-d}) \cong KO(\mathbf{S}^{p-d}), \quad (3.2)$$

and it is valid for all p . Hence, this is the relevant group that classify wrapped $D(d+9-p)$ -branes, in the ten dimensional theory, on \mathbf{T}^{9-p} and on top of an Op -plane. From now on, we will refer to these branes as the T-dual version of branes on Type I (or $USp(32)$) theories until we require to be more specific. For Op^+ -planes, T-dual version of branes in $USp(32)$ string theory, are given by the quaternionic K-theory group,

$$KH^{p-9}(\mathbb{R}^{9-p,p-d}) \cong KSp(\mathbf{S}^{p-d}) \cong KO(\mathbf{S}^{p-d+4}). \quad (3.3)$$

In the next section we will compute RR fields associated to these kind of branes.

3.2 Discrete charges and global gauge anomalies

Anomalies in probe branes on compact spaces are related to non-zero RR discrete charges and with the presence of \widetilde{Op}^\pm -planes for $p > 6$ [25, 6].

We are interested on global gauge anomalies [26] arising in intersecting probe D-branes with discrete charge on compact spaces. In [25] it was shown that by using D5

⁷Actually T-duality acts on derived categories, or roughly speaking, on K-theory. See Ref. [24].

probe branes wrapped on \mathbf{T}^2 in Type I theory, that $\widehat{\text{D7}}$ -branes should exist in an even number in order that global gauge anomalies be canceled, i.e., discrete K-theory charge should be canceled.

The idea is to consider $2n$ coincident D5-branes wrapped on \mathbf{T}^2 and one $\widehat{\text{D7}}$ sitting at a point in \mathbf{T}^2 . The four dimensional non-compact space that intersects the $\widehat{\text{D7}}$ -brane contains fields arising from strings attached to both branes. Since $\widehat{\text{D7}} = \text{D7} + \overline{\text{D7}}/\Omega$ from IIB theory, it is enough to compute the sector 75 and $\bar{75}$. The result is the existence of a four-dimensional Weyl fermion in the fundamental representation $2n$ of $USp(2n)$. This gives rise to a $SU(2)$ global gauge anomaly [26]. The argument can be extended to orientifolds $\mathbf{T}^4/\mathbf{Z}_2$ in the IIB theory.

Now we want to show that also $\widehat{\text{D4}}$ - and $\widehat{\text{D3}}$ -branes in $USp(32)$ string theory give rise to global gauge anomalies on suitable probe D-branes. We consider a compactification of $USp(32)$ theory on \mathbf{T}^6 with a single $\widehat{\text{D3}}$ -brane extending along the four non-compact dimensions, and placing it at a point in \mathbf{T}^6 . These systems contain tachyonic modes arising from the $\widehat{39}$ and $\bar{93}$ open string sectors. The $\widehat{\text{D3}}$ -brane carries a \mathbf{Z}_2 -charge measured by K-theory. In this case, the suitable probe branes are the $\bar{9}$ -branes themselves (remember that there are $\overline{\text{D9}}$ -branes because tadpole cancellation in $USp(32)$ string theory). $\widehat{\text{D3}}$ -brane is constructed in string theory as a Type IIB $\text{D3}-\overline{\text{D3}}$ -pair exchanged by Ω . Let us compute the nonsupersymmetric spectrum arising from $\widehat{39}$ and $\bar{93}$ sectors. Sectors $\bar{93}$ and $\bar{93}$ are mapped into $\bar{39}$ and 39 . In the fermionic content there is a Weyl fermion in the fundamental representation $2n$ of $USp(2n)$. This is inconsistent at the quantum level. Thus, the $\widehat{\text{D3}}$ -branes should appear in pairs on compact spaces.

We conclude that for $\widehat{D}(p-6)$ -branes on top of an Op^+ -plane (with $p = 5, 6$) also must be in pairs. The same result is valid for the $\widehat{\text{D4}}$ in $USp(32)$ string theory and for $\widehat{D}(p-5)$ -branes (for $p = 4, 5, 6$) on T-dual versions of $USp(32)$ theory with Op^+ -planes.

4 RR Fields, Orientifolds and K-theory

The aim of this section is to classify RR fields in the presence of orientifold planes and branes on top of them. The procedure is as follows: firstly, we give a briefly review of K-theory classification of RR fields in type II string theories; this survey is based in [18]. Secondly, we review the K-theory classification of RR fields with orientifolds, which was studied in [5]. Finally, we take the results given in section 3 and the K-theory classification of RR fields with orientifolds (given in the present section) in order to obtain the K-theory classification of RR fields in the presence of orientifold planes

and d -branes (on top of the Op -planes). As it is shown below, we get a K-theory group which classifies RR fields in such backgrounds.

4.1 RR fields and K-theory

It is well known that D-brane charges are classified by K-theory rather than by cohomology. Recently it was shown that also RR fields are classified by K-theory [18], even though they are not related to a source. Let us remind this important fact.

It is possible to show that RR charge is measured by the kernel of the map $i : K(\mathcal{M}, \mathcal{N}; \mathbb{Z}) \rightarrow K(\mathcal{M}; \mathbb{Z})$, with $K(\mathcal{M}, \mathcal{N}; \mathbb{Z})$ being the K-theory group which classifies classes of bundles on \mathcal{M} that are trivial on \mathcal{N} , where \mathcal{M} is the spacetime manifold and \mathcal{N} is its boundary. The important fact is that

$$\text{Ker}(i) = K^1(\mathcal{N})/j(K^1(\mathcal{M})), \quad (4.1)$$

with j the restriction to the boundary $\mathcal{N} = \partial\mathcal{M}$, and where $K^1(\mathcal{N})$ classifies RR fields at infinity and $K^1(\mathcal{M})$ classifies fields on \mathcal{M} that *do not have any brane source* (in Type IIB theory), i.e., the K-theory classification of RR charges is given by the group $K(\mathcal{M})$, while RR fields are classified by $K^1(\mathcal{M})$.

The result is easily extended to Type IIA and Type I theories. The groups are $K(\mathcal{M})$ and $KO^{-1}(\mathcal{M})$ respectively.

4.2 Real K-theory and orientifold classification

Although we have seen a cohomological classification of orientifold planes, there are some issues that cohomology is not able to explain. For instance, when discrete fluxes are turned on, the charges and tensions of orientifold planes are changed, giving rise to different types of planes for the same dimensionality, as it was seen in section 2. For instance, the charge of Op^- differs with respect to \widetilde{Op}^- by one half (in D-brane units of charge). This issue is not explained by cohomology. However, by using K-theory, Bergman, Gimon and Sugimoto (BGS) [5] explained the relative charge between the above orientifolds, and moreover, they found some new correlations among other types of orientifold planes. This was done by the K-theory classification of RR fields in the presence of orientifold planes.

The K-theory groups which classify RR fields in orientifold backgrounds, according to BGS, are given by

$$\begin{aligned} Op^- : & \quad KR^{p-10}(\mathbf{S}^{9-p,0}), \\ Op^+ : & \quad KR^{p-6}(\mathbf{S}^{9-p,0}) = KH^{p-10}(\mathbf{S}^{9-p,0}). \end{aligned} \quad (4.2)$$

They are easily calculated by using the Atiyah isomorphism

$$KR^{-n}(\mathbb{S}^{p,0} \times X) = KR^{p-n+1}(X) \oplus KR^{-n}(X), \quad (4.3)$$

with $X = \{pt\}$ and by knowing the groups for a point space, which read

$$KR^{-n}(\{pt\}) = \{\mathbb{Z}, \mathbf{Z}_2, \mathbf{Z}_2, 0, \mathbb{Z}, 0, 0, 0\} \bmod 8. \quad (4.4)$$

An example: The $O5$ -plane. Let us explain some important details of this classification by analyzing one specific example: the orientifold five-plane. The cohomology classification of this orientifold, as we saw in section 2, is given by the groups $H^3(\mathbb{RP}^3, \mathbb{Z}) = \mathbb{Z}$ (which give us the integer RR charge of D5-branes on top of it), and $H^1(\mathbb{RP}^3; \widetilde{\mathbb{Z}}) = \mathbf{Z}_2$ (the non-trivial element of \mathbf{Z}_2 give us the existence of the orientifold variant $\widetilde{O5}^\pm$). Notice that we are classifying orientifolds according to the cohomology group of RR fields, that is the reason why we have actually one single group for the two variants $O5^\pm$, which means that a cohomology of RR forms does not distinguish between $O5^+$ and $O5^-$ planes. Now, according to the above results, the K-theory classification of orientifold five-planes is given by the groups $KR^{-5}(\mathbb{S}^{4,0}) = \mathbb{Z}$ for $O5^-$ and $KH^{-5}(\mathbb{S}^{4,0}) = \mathbb{Z} \oplus \mathbf{Z}_2$ for $O5^+$. Notice that these groups are classifying RR fields and that give us a different result as the RR charges classification (which just give us the value of \mathbb{Z} for both cases). So, there are some important questions to address: what does this difference between RR fields and RR charges K-theory classification mean? and what does the difference between cohomology and K-theory means? Both of them were correctly answered by BGS. The answer for the first question is that there are RR fields not related to D-branes but to the presence of orientifold planes, i.e., discrete RR fields are turned on by placing orientifold planes in the background. That is the reason why a K-theory classification of RR fields is in fact, an orientifold classification. The answer for the second question involves a feature which has been well studied by mathematicians. Indeed, there is an algorithm which relates cohomology to K-theory and that, also gives some physical consequences when it is applied to the above case. Hence, before we continue describing the case of the orientifold five-plane, let us review this algorithm called the Atiyah-Hirzebruch Spectral Sequence (AHSS).

4.2.1 The Atiyah-Hirzebruch Spectral Sequence (AHSS)

The AHSS is an algebraic algorithm that allows to relate K-theory to integral cohomology (see for instance, [5, 22, 27]).

The basic idea of the AHSS is to compute $K(X)$ using a sequence of successive approximations, starting with integral cohomology⁸. Basically each step of approximation

⁸For an introductory review of the AHSS see [5] and references therein. Also see [29].

is given by the cohomology of a differential operator d_r , denoted as

$$E_{r+1}^p = \ker d_r / \text{Im } d_r^{p-r}, \quad (4.5)$$

where $d_r^p : E_r^p \rightarrow E_r^{p+r}$. In each step, we refine the approximation by removing cohomology classes which are not closed under the differential d_r^p . Closed classes survive the refinement while exact classes are mapped to trivial ones in the next step. In the complex case (without orientifolds), the first non-trivial higher differential is given by $d_3 = Sq^3 + H_{NS}$, where Sq^3 is the Steenrod square and H_{NS} is the NS-NS three form. In the case of string theory, the only possible next higher differential is d_5 .

By the above procedure we get the associated graded complex $GrK(X)$ which is the approximation to $K(X)$. The graded complex is given by

$$GrK(X) = \oplus_p E_r^p = \oplus_p K_p(X) / K_{p+1}(X) \quad (4.6)$$

where $K_n(X) \subset K_{n-1}(X) \subset \dots \subset K_0(X) = K(X)$. At the first approximation we have

$$K_p(X) / K_{p+1}(X) = \begin{cases} H^p(X, \mathbb{Z}) & \text{for } p \text{ even} \\ 0 & \text{for } p \text{ odd} \end{cases} \quad (4.7)$$

for Type IIA theory, and

$$K_p(X) / K_{p+1}(X) = \begin{cases} H^p(X, \mathbb{Z}) & \text{for } p \text{ odd} \\ 0 & \text{for } p \text{ even} \end{cases} \quad (4.8)$$

for Type IIB. Thus, computing $K(X)$ implies that we have to resolve the follow extension problem,

$$0 \longrightarrow K_{p+1}(X) \longrightarrow K_p(X) \longrightarrow K_p(X) / K_{p+1}(X) \longrightarrow 0. \quad (4.9)$$

If the above sequence is trivial we have that

$$K_p(X) = K_{p+1}(X) \oplus K_p(X) / K_{p+1}(X). \quad (4.10)$$

If all extensions are trivial, then $K(X) = GrK(X)$. In our case, we just have to worry about the mapping d_3 . If d_3 is trivial we finish at the cohomology level, and we must ask about the exactness of the sequence. When the sequence is not exact, p -forms of different degree become correlated and physically this means that we have correlations between the associated RR fields.

For real K-theory (or in general, for K-theory groups with freely acting involutions) the approximations are given by *twisted* or *untwisted* maps (see appendix in [5]), i.e.,

the d_3 differential operator maps twisted into untwisted classes and vice versa. In this case $d_3 = \widetilde{Sq^3} + H_{NS}$, with $[H_{NS}] \in \mathbf{Z}_2$. It is assumed that $\widetilde{Sq^3}$ is trivial for both values of \mathbf{Z}_2 (i.e., for Op^+ and Op^-) and d_5 is trivial in all cases. d_5 maps (un)twisted into (un)twisted classes.

The first approximation to the graded complex $GrK^{-s}(X) = \bigoplus_n E_n^{p,-(p+s)}$, with

$$E_n^{p,-(p+s)}(X) = K_p^{-s}(X)/K_{p+1}^{-s}(X), \quad (4.11)$$

is given by

$$\begin{aligned} E_1^{p,q} &= C^p(X|_\tau, \mathbb{Z}) \text{ for } q = 0 \bmod 4 \\ E_1^{p,q} &= C^p(X|_\tau, \widetilde{\mathbb{Z}}) \text{ for } q = 2 \bmod 4 \\ E_1^{p,q} &= 0 \text{ for } q \text{ odd,} \end{aligned} \quad (4.12)$$

where τ is the freely acting involution on X . Then, the second order of this approximation is given by the cohomology groups

$$\begin{aligned} E_2^{p,q} &= H^p(X|_\tau, \mathbb{Z}) \text{ for } q = 0 \bmod 4 \\ E_2^{p,q} &= H^p(X|_\tau, \widetilde{\mathbb{Z}}) \text{ for } q = 2 \bmod 4 \\ E_2^{p,q} &= 0 \text{ for } q \text{ odd.} \end{aligned} \quad (4.13)$$

The same results stand for quaternionic K-theory groups.

It is important to point out that triviality of Steenrod square which is also taken in the untwisted version, actually has a physical interpretation. $Sq^3 = 0$ implies that $W_3(Q) = 0$, where W_3 is the Bockstein homomorphism and the above relation expresses the fact that a D-brane can be wrapped on a submanifold Q . This means that Q must be a Spin_c manifold [10]. When the NS H -field is different from zero, the required topological condition is

$$[H_{NS}] + W_3(Q) = 0. \quad (4.14)$$

It is shown in [28] that this is in fact the condition to cancel anomalies arising in the worldsheet of strings in the presence of D-branes in Type II theory. On the other hand, the AHSS described in terms of branes (see appendix of [27]), requires to wrap D-branes on submanifolds Q . Thus, in order to lift trivially cohomology forms to K-theory, we need that $d_3 = 0$, or that suitable D-branes wrap on Spin_c manifolds. When this submanifold is not Spin_c cohomology and K-theory differ from each other.

Now, for the twisted version of d_3 it is assumed the same triviality in the twisted version of the Steenrod square. This means that a topological condition could be

also present for the case of Type I theory and then, there is an anomaly present in the worldsheet of open strings in the presence of D-branes and orientifolds of Type II theories. It would be interesting to study what could be the ‘twisted’ version of a Spin_c manifold.

Example: The $O5$ -plane

Once we have a procedure to compare or lift cohomology to K-theory, and by knowing the K-theory groups which classify RR fields related to orientifold planes, it is possible to get a physical picture which interprets the difference between cohomology and K-theory. Let us come back to our example of the orientifold five plane. In this case d_3 is trivial for both types of $O5$ -planes, as well as d_5 . Hence, the approximation ends at cohomology. It is possible to show that the extension problem to solve is

$$\begin{array}{ccccccc}
0 & \longrightarrow & \mathbb{Z} & \xrightarrow{\begin{cases} \times 2 & \text{for } O5^- \\ Id & \text{for } O5^+ \end{cases}} & \begin{cases} \mathbb{Z} & \text{for } O5^- \\ \mathbb{Z} \oplus \mathbb{Z}_2 & \text{for } O5^+ \end{cases} & \longrightarrow & \mathbb{Z}_2 & \longrightarrow & 0 \\
& & \parallel & & \parallel & & \parallel & & (4.15) \\
& & H^3 & & \begin{cases} KR^{-5}(\mathbb{S}^{4,0}) & \text{for } O5^- \\ KH^{-5}(\mathbb{S}^{4,0}) & \text{for } O5^+ \end{cases} & & \widetilde{H}^1 & &
\end{array}$$

In the case of the $O5^+$ -plane, the sequence is trivial while for the case of $O5^-$ it is not. In the latter case this means that a half-integer shift is produced in H^3 due to the presence of the flux $G_1 \in H^1$. The physical implication is as follows: cohomology gives us a classification of orientifolds that must be refined by K-theory. The refinement is produced by the half-integer shift in the flux G_3 , or in other words, by a half-integer shift in the RR charge of the orientifold $O5^-$. Afterwards, the K-theory picture, through the application of the AHSS, explain why the $\widetilde{O5}^-$ -plane has precisely, an extra half-integer amount of RR charge than the ordinary $O5^-$ -plane. So, an $\widetilde{O5}^-$ can be written as $O5^- + \frac{1}{2}\widehat{D5}$. The same description holds for all the lower orientifolds \widetilde{Op}^- . In the case of an $O5^+$ plane there is no an extra shift in the RR charge of $\widetilde{O5}^+$, and then its charge is the same than an $O5^+$ -plane. This case is trivial and cohomology gives an exact description of the K-theory group (the graded complex is equal to the K-theory group). The anti-D5-brane on top of the $O5^+$ -plane corresponds to a stable but non-supersymmetric system [30].

Another interesting result involves the $O3$ -plane. In such a case, the approximation given by the AHSS, does not ends at the first step, since d_3 is not trivial for $O3^+$ (for Op^- , d_3 is always trivial since $H_{NS} = 0$ and the twisted version of Sq^3 is trivial as well). Hence, the non-trivial discrete class of $H^3(\mathbb{RP}^5; \widetilde{\mathbb{Z}})$ (which at the cohomology

level suggests the presence of an $\widetilde{O3}^+$ -plane) is obstructed to be lifted to K-theory (it is not a closed form under d_3). The conclusion is that both orientifolds, $O3^+$ and $\widetilde{O3}^+$, are actually the same object.

4.3 Branes, orientifolds and K-theory

We have seen that RR fields are classified by K-theory even if they are source-free. Also that this feature allow us to classify RR-fields in orientifold backgrounds and to find some correlations between Op^\pm -planes.

Now we are interested in classifying RR fields in the presence of orientifold planes, and d -branes, with $d < p$. We expect to obtain RR fields associated to Dd -branes (T-dual versions of those D-branes living on Type I and $USp(32)$ string theories) present on top of an Op^\pm -plane, i.e., with all their coordinates along the orientifold (they are in the set of RR charges classified by K-theory). We are also interested in classifying RR fields that are not associated to the above D-branes, and which in turn be discrete fluxes in the background.

In order to classify these fields, we have to answer first some questions:

1. Which is the K-theory group that classifies RR fields in the presence of d -branes and Op -planes?
2. If we want to find charge correlations, as was done in [5] for the orientifolds, we must use the AHSS. But this requires the knowledge of (related) cohomology groups. Thus, which are the relevant cohomology groups classifying RR forms with d -branes and Op -planes?
3. If there are RR fields without a source in the presence of orientifold and d -branes, what is the role played by d -branes associated to such fields?

Let us start by answering the first question. For that, we need to describe how to wrap D8-branes on spacetime in order to know which K-theory groups are the relevant ones to classify RR fields in the mentioned conditions. In Ref. [5] a D8-brane was wrapped on a S^{8-p} sphere on the covering transverse space \mathbb{R}^{9-p} . After taking the orientifold action, the transverse space is \mathbb{RP}^{8-p} . Hence, actually one is wrapping a D8-brane on $S^{9-p,0}$, which is the unitary sphere on $\mathbb{R}^{9-p,0}$. By this procedure, BGS get the K-theory groups given in Eq. (4.2).

Now we want to wrap 8-branes on the transverse space to a d -brane on top of an Op -plane, for $d < p$. Then we wrap 8-branes on

$$S^{8-p} \times \mathbb{R}^{p-d} \tag{4.16}$$

in the covering space. Two comments are in order. First, note that the space \mathbb{R}^{p-d} is transverse to the d -brane but it is still immersed in the Op -plane. Second, for $p = d$ we recover BGS results [5]. The above product of spaces can be written as

$$\mathbb{S}^{9-p,0} \times \mathbb{R}^{0,p-d}. \quad (4.17)$$

Now, in order to show that the K-theory groups, which classify RR fields for Op -planes, do not change their order for a fixed value of p , in relation to the suitable groups given in 4.2, consider the $O8$ -projection $\mathbb{R}^1/\mathcal{I}_1\Omega$ in Type IIA theory. $O8$ maps a D8 wrapping a point on one side of the orientifold to an image D8 wrapping the other point (looks like a wrapped $\overline{D8}$). So the relevant K-theory group is KR_{\pm} for Type I' (with two $O8^-$) and KH_{\pm} for the T-dual version of $USp(32)$ (with two $O8^+$).

Now by wrapping a D8-brane on a $\mathbb{S}^0 \times \mathbb{R}^1$ (taking for instance $d = 7$), the transverse space to a 7-brane inside an $O8$ -plane is actually divided into two parts. Since the fraction of the D8 wrapped on \mathbb{R}^1 is on the orientifold, it is its own self-image since its orientation can be regarded as the orientation of an anti-brane with reversal orientation. Hence, repeating this procedure for all Op -planes we conclude that K-theory groups must be exactly the same than those given by BGS, but over different suitable spaces. In other words, \mathbb{R}^{p-d} is fixed under the orientifold projection.

Hence, K-theory groups that classify RR fields on an orientifold and Dd -branes backgrounds are:

$$\begin{aligned} Op^- : & \quad KR^{p-10}(\mathbb{S}^{9-p,0} \times \mathbb{R}^{0,p-d}), \\ Op^+ : & \quad KR^{p-6}(\mathbb{S}^{9-p,0} \times \mathbb{R}^{0,p-d}). \end{aligned} \quad (4.18)$$

Using the Atiyah isomorphism [15], we get

$$\begin{aligned} KR^{-n}(\mathbb{S}^{m,0} \times \mathbb{R}^{0,l}) &= KR^{-n+m+1}(\mathbb{R}^{0,l}) \oplus KR^{-n}(\mathbb{R}^{0,l}) \\ &= KR^{-n+m+1,l}(\{pt\}) \oplus KR^{-n,l}(\{pt\}) \\ &= KR^{-n+m-l+1}(\{pt\}) \oplus KR^{-(n+l)}(\{pt\}) \\ &= KR^{-(n+l)}(\mathbb{S}^{m,0} \times \{pt\}). \end{aligned} \quad (4.19)$$

Replacing the variables by taking

$$\begin{aligned} -n &\rightarrow p - 10 \\ m &\rightarrow 9 - p \\ l &\rightarrow p - d, \end{aligned} \quad (4.20)$$

we get our final expression that allow us to calculate RR fluxes on a d -dimensional submanifold within the orientifold Op^- , or the RR fluxes related to d -branes on top of orientifold planes. For Op^+ -planes we have similar results⁹:

⁹The following expressions are just valid for $p \leq 6$. For $p > 6$ we have the usual results, i.e., the second term of the right-hand side is not present since Atiyah isomorphism is not longer valid.

Table 1: RR fluxes for d -branes on top of Op^- -planes.

d	$O8^-$	$O7^-$	$O6^-$	$O5^-$	$O4^-$	$O3^-$	$O2^-$	$O1^-$	$O0^-$
8	\mathbb{Z}								
7	\mathbf{Z}_2	\mathbb{Z}							
6	\mathbf{Z}_2	\mathbf{Z}_2	$\mathbb{Z} \oplus \mathbb{Z}$						
5	0	\mathbf{Z}_2	$\mathbf{Z}_2(\oplus 0)$	$\mathbb{Z}(\oplus 0)$					
4	\mathbb{Z}	0	$\mathbf{Z}_2(\oplus 0)$	$\mathbf{Z}_2(\oplus 0)$	$\mathbb{Z}(\oplus 0)$				
3	0	\mathbb{Z}	0	$\mathbf{Z}_2(\oplus 0)$	$\mathbf{Z}_2(\oplus 0)$	$\mathbb{Z}(\oplus 0)$			
2	0	0	$\mathbb{Z} \oplus \mathbb{Z}$	$(0 \oplus) \mathbb{Z}$	$\mathbf{Z}_2 \oplus \mathbb{Z}$	$\mathbf{Z}_2 \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z}$		
1	0	0	$(0 \oplus) \mathbf{Z}_2$	$\mathbb{Z} \oplus \mathbf{Z}_2$	$(0 \oplus) \mathbf{Z}_2$	$\mathbf{Z}_2 \oplus \mathbf{Z}_2$	$\mathbf{Z}_2 \oplus \mathbf{Z}_2$	$\mathbb{Z} \oplus \mathbf{Z}_2$	
0	\mathbb{Z}	0	$(0 \oplus) \mathbf{Z}_2$	$(0 \oplus) \mathbf{Z}_2$	$\mathbb{Z} \oplus \mathbf{Z}_2$	$(0 \oplus) \mathbf{Z}_2$	$\mathbf{Z}_2 \oplus \mathbf{Z}_2$	$\mathbf{Z}_2 \oplus \mathbf{Z}_2$	$\mathbb{Z} \oplus \mathbf{Z}_2$
(-1)	\mathbf{Z}_2	\mathbb{Z}	0	0	0	$\mathbb{Z}(\oplus 0)$	0	$\mathbf{Z}_2(\oplus 0)$	$\mathbf{Z}_2(\oplus 0)$

$$\begin{aligned}
Op^- : \quad KR^{d-10}(\mathbb{S}^{9-p,0}) &= KR^{d-p}(\{pt\}) \oplus KR^{d-10}(\{pt\}) , \\
Op^+ : \quad KR^{d-6}(\mathbb{S}^{9-p,0}) &= KR^{d-p+4}(\{pt\}) \oplus KR^{d-6}(\{pt\}) .
\end{aligned} \tag{4.21}$$

The results of the computation of these groups for Op^- and Op^+ planes are summarized in tables 1 and 2 respectively. Notice that for the case $d = p$ we recover BGS results [5]. Once we have calculated these groups many interesting issues result from it. In the next sections we will describe some of them.

5 (Co)homology and D-branes in Orientifolds

Before interpreting physically the RR fields shown in tables 1 and 2 we must answer the second question raised in the previous section: What is the cohomology groups which classify RR fields in the presence of d -branes and Op -planes (for $d \leq p$)?

In this section we give the answer by wrapping $D(d+n)$ -branes on homology n -cycles. To obtain the cohomology groups we first find their associated homology groups and by Poincaré duality we can find them. Our aim is to compare these results to those obtained by K-theory in the previous section by using the AHSS. This will be the goal of the next section.

Table 2: RR fluxes for d -branes on top of Op^+ -planes.

d	$O8^+$	$O7^+$	$O6^+$	$O5^+$	$O4^+$	$O3^+$	$O2^+$	$O1^+$	$O0^+$
8	\mathbb{Z}								
7	0	\mathbb{Z}							
6	0	0	$\mathbb{Z} \oplus \mathbb{Z}$						
5	0	0	$(0 \oplus) \mathbf{Z}_2$	$\mathbb{Z} \oplus \mathbf{Z}_2$					
4	\mathbb{Z}	0	$(0 \oplus) \mathbf{Z}_2$	$(0 \oplus) \mathbf{Z}_2$	$\mathbb{Z} \oplus \mathbf{Z}_2$				
3	\mathbf{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}			
2	\mathbf{Z}_2	\mathbf{Z}_2	$\mathbb{Z} \oplus \mathbb{Z}$	$(0 \oplus) \mathbb{Z}$	$(0 \oplus) \mathbb{Z}$	$(0 \oplus) \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z}$		
1	0	\mathbf{Z}_2	$\mathbf{Z}_2(\oplus 0)$	$\mathbb{Z}(\oplus 0)$	0	0	0	\mathbb{Z}	
0	\mathbb{Z}	0	$\mathbf{Z}_2(\oplus 0)$	$\mathbf{Z}_2(\oplus 0)$	$\mathbb{Z}(\oplus 0)$	0	0	0	\mathbb{Z}
(-1)	0	\mathbb{Z}	0	$\mathbf{Z}_2(\oplus 0)$	$\mathbf{Z}_2(\oplus 0)$	\mathbb{Z}	0	0	0

5.1 Wrapping D-branes on homological cycles

Wrapping $D(p+n)$ -branes in non-trivial and compact homology n -cycles of projective spaces¹⁰ has been used extensively to classify fluxes which give rise to different kind of orientifold planes [4, 5, 6].

As we have seen in previous sections, new types of orientifold planes (\widetilde{Op}^\pm) appear when discrete RR fluxes are turned on. These fluxes are classified by the cohomology of projective spaces (the transversal spaces to the orientifolds), i.e., by the group $H^{6-p}(\mathbb{RP}^{8-p}; \mathbb{Z}(\widetilde{\mathbb{Z}}))$. The “brane realization of RR discrete torsion” is obtained by intersecting a $D(p+2)$ -brane and an Op^\pm -plane at one point. Then, it is possible to deform the D-brane in such a way that it wraps on a two-cycle of \mathbb{RP}^{8-p} . If the origin is not removed, the two-cycle is not a truly homological cycle of the bulk space and it shrinks to a point, giving rise to our original configuration of a $D(p+2)$ -brane intersecting the Op^\pm -plane. However, if the origin is removed, we actually are allowed to wrap $D(p+2)$ -branes on homological two-cycles of \mathbb{RP}^{8-p} to get an \widetilde{Op}^\pm -plane. Moreover, according to Ref. [6], a $D(p+2)$ -brane wrapped on a two-cycle carries a RR charge of a Dp -brane. So, what we have indeed, is that after the wrapping process, we get a truly Dp -brane where the RR charge is given directly by the value of the (co)homology group which classifies the two-cycle wrapped by the $D(p+2)$ -brane.

¹⁰In fact, they are not truly homological cycles in the bulk space to the orientifold, unless we are removing the origin. In this construction, we remove the origin in order to obtain stable branes by wrapping them on non-trivial homological cycles [29].

This is indeed the idea we want to use in order to obtain the cohomology groups which classify RR fields in the presence of orientifold planes and lower dimensional branes on top of them.

5.1.1 Op -planes and p -branes

Let us start by *re-obtaining* the cohomology groups which classify orientifolds. According to the “brane realization picture of discrete RR fluxes”, we must take a $D(p+2)$ -brane and wrap it on a two-cycle of \mathbb{RP}^{8-p} (notice that this cycle can be twisted or untwisted). However, it turns out that the non-zero valued homological group classifying two-cycles is $H_2(\mathbb{RP}^{8-p}; \tilde{\mathbb{Z}}) = \mathbf{Z}_2$, which actually classifies *twisted* cycles. The fact that we require twisted cycles, can also be understood from a physical perspective: $D(p+2)$ -branes couple with $(p+3)$ -forms in the bulk space, and according to relations (2.7), these forms are in fact, *twisted*. So, a $D(p+2)$ -brane can only be wrapped on twisted cycles. Finally, by using Poincaré duality, which reads,

$$\begin{aligned} \text{For } n \text{ odd: } H_i(\mathbb{RP}^n; \mathbb{Z}(\tilde{\mathbb{Z}})) &\cong H^{n-i}(\mathbb{RP}^n; \mathbb{Z}(\tilde{\mathbb{Z}})), \\ \text{For } n \text{ even: } H_i(\mathbb{RP}^n; \mathbb{Z}(\tilde{\mathbb{Z}})) &\cong H^{n-i}(\mathbb{RP}^n; \tilde{\mathbb{Z}}(\mathbb{Z})), \end{aligned} \tag{5.1}$$

we find that the cohomology group which classifies orientifold planes (when discrete RR fluxes are turned on) is actually $H^{6-p}(\mathbb{RP}^{8-p}; \tilde{\mathbb{Z}}) = \mathbf{Z}_2$. Notice that, according to this procedure, the above cohomology group is also *the one which classifies RR fields in the presence of Op^\pm -planes and p -branes*. Let us fix the notation: a p -brane stands for a generic p -dimensional brane, while one with discrete \mathbf{Z}_2 topological charge, will be denoted as a \hat{p} -brane. This notation stands from the fact that up to this point we do not the nature of these objects. We require K-theory in order to get a more precise description of them.

Thus, we can get (as a first approximation) a picture of an \widetilde{Op}^\pm -plane as one Op^\pm -plane plus a \hat{p} -brane. Of course, this turns out to be not correct at all, since an \widetilde{Op}^- -plane is given by an Op^- plus a half stuck brane, $\frac{1}{2}Dp$. Notice also that this description is valid just for the case $2 \leq 8-p$, i.e., for $p \leq 6$.

There is a second possibility to get a p -brane (or a \hat{p} -brane) by wrapping D-branes on homology cycles. This is given by wrapping a $D(p+6)$ -brane on a 6-cycle of \mathbb{RP}^{8-p} . The homology group classifying such cycles is $H_6(\mathbb{RP}^{8-p}) = \mathbf{Z}_2$. By Poincaré duality this is the cohomology group $H^{2-p}(\mathbb{RP}^{8-p}) = \mathbf{Z}_2$ which actually classifies other type of orientifold planes denoted as \widehat{Op}^\pm . Notice that this is possible just for the case $0 \leq 2-p$, i.e., for $p \leq 2$. By the same argument as above, the exotic orientifold plane \widehat{Op}^\pm can be expressed as the sum of an Op^\pm -plane plus a \hat{p} -brane and moreover,

for $p \leq 2$ we actually have 8 different types of orientifold planes by taking all the possible combination of RR discrete fluxes. Hence, our description of wrapping branes on homology cycles reproduce all these well-known results. Our goal, for the next section is to describe the nature of the \widehat{p} -branes and establish a difference between the ones associated to \widetilde{Op}^\pm and \widehat{Op}^\pm .

From the above analysis we get two important results: 1) we have a procedure to classify by cohomology all the spectrum of RR fields in the presence of orientifolds Op^\pm and d -branes, and 2) we require to classify them by K-theory in order to *refine* our conclusion of what an \widetilde{Op}^\pm -plane or an \widehat{Op}^\pm -plane are made of.

5.1.2 Op -planes and d -branes

Let us start by working out the point 1). Our goal is to extend this idea to any d -brane on top of an Op -plane, with $d < p$. This means that we will be able (in the cohomology sense) to obtain d -branes by wrapping $D(d+n)$ -branes on non-trivial compact homological n -cycles¹¹. In order to do that, we require to know what homological cycles are suitable for wrapping D-branes on them, as was done for the $O6$ -plane in Ref. [31]. The answer is given by the relations (2.7).

Far away from the orientifold plane and locally, the relevant string theory is the Type II one (A or B depending of the dimension of the orientifold plane). The RR forms C_{d+n+1} couple to $D(d+n)$ -branes, and they are affected by the orientifold projection as in equations (2.7). According to the nature of the RR form, twisted or untwisted, the associated brane can be wrapped on a homological cycle of the same nature, i.e., a brane which couples to a (un)twisted form, can be wrapped only on a (un)twisted cycle. The RR Dd -brane charge of a $D(d+n)$ -brane wrapped on a non-trivial homological n -cycle is the same that the corresponding n -th homology group value of \mathbb{RP}^{8-p} . Finally, by Poincaré duality, we can obtain the relevant cohomology group for such d -branes.

An example: The cohomology of $O5$.

In order to give an specific example, take for instance the $O5$ -plane. By Eq. (2.7) we know that $D7$, $D3$ and $D(-1)$ -branes for Type IIB theory can be wrapped only on twisted cycles and $D9$, $D5$ and $D1$ -branes on untwisted ones. Then, the homology

¹¹We are restricting ourselves to the study of D-branes completely immersed in the orientifold plane.

Table 3: The table shows the twisted and untwisted n -cycles in where suitable $D(d+n)$ -branes can be wrapped.

Op -plane	On untwisted cycles	On twisted cycles
$p = 6, 2$	D6,D2	D8,D4,D0
$p = 5, 1$	D9,D5,D1	D7,D3,D(-1)
$p = 4, 0$	D8,D4,D0	D6,D2
$p = 3$	D7,D3,D(-1)	D9,D5,D1

groups of \mathbb{RP}^3 are given by

$$\begin{aligned}
H_0(\mathbb{RP}^3, \mathbb{Z}) &= \mathbb{Z}, & H_0(\mathbb{RP}^3, \widetilde{\mathbb{Z}}) &= \mathbf{Z}_2, \\
H_1(\mathbb{RP}^3, \mathbb{Z}) &= \mathbf{Z}_2, & H_2(\mathbb{RP}^3, \mathbb{Z}) &= \mathbf{Z}_2, \\
H_3(\mathbb{RP}^3, \mathbb{Z}) &= \mathbb{Z}.
\end{aligned}$$

Now, by wrapping $D(d+n)$ -branes (with $0 \leq n \neq 3$) we obtain the desired d -branes. For instance, wrapping D3 and D(-1)-branes on the twisted 0-cycle we obtain states that are identified with $\widehat{3}$ - and $\widehat{(-1)}$ -branes (since the 0-cycle has \mathbf{Z}_2 -charge). If now we wrap D7 and D3 on twisted two-cycles we get $\widehat{5}$ and $\widehat{1}$ -branes. On the other hand, wrapping D5 and D1 branes on untwisted 0, 1 and 3-cycles, we obtain 5 and 1, $\widehat{4}$ and $\widehat{(-1)}$, and 2 branes respectively.

For completeness and future reference, we proceed similarly for all orientifolds Op with $p \leq 6$. The results are listed in tables 3 and 4.

As we can see from these tables, in general there are three different types of cohomology groups classifying RR forms in the presence of Op -planes and d -branes. This is as follows:

- $H^{8-p}(\mathbb{RP}^{8-p}) = \mathbb{Z}$. This group classify RR forms related to Dp -branes on top of Op^\pm -planes. Give us the usual integer RR charge of such branes.
- $H^{6-d}(\mathbb{RP}^{8-p}) = \mathbf{Z}_2$. It classifies RR forms related to \widehat{d} -branes on top of Op -planes. Notice that in the case $d = p$, the non-trivial class of \mathbf{Z}_2 stands for the presence of an \widetilde{Op}^\pm -plane. This is true for $p - d \leq 2$ and $d \leq 6$.
- $H^{2-d}(\mathbb{RP}^{8-p}) = \mathbf{Z}_2$ ($d \neq 2$). It also classifies RR forms related to \widehat{d} -branes. Notice that in the case $d = p$, we recover the classification of \widehat{Op}^\pm -planes. This case is valid just for $d < 2$ and $p - d \leq 6$. In the case of $d = 2$ the cohomology value is integer, and it is related to d -branes.

Table 4: Dd -branes obtained by wrapping $D(d+n)$ -branes on n -cycles. We label as Dd -branes the branes which are also classified by K-theory. The other ones are labeled just as d -branes.

Op -planes	$H_n(\mathbb{RP}^{8-p}; \mathbb{Z})$	Dd -branes	$H_n(\mathbb{RP}^{8-p}; \widetilde{\mathbb{Z}})$	Dd -branes
6	$H_0(\mathbb{RP}^2; \mathbb{Z}) = \mathbb{Z}$ $H_1(\mathbb{RP}^2; \mathbb{Z}) = \mathbf{Z}_2$	D6 D2 $\widehat{D5}$ $\widehat{D1}$	$H_0(\mathbb{RP}^2; \widetilde{\mathbb{Z}}) = \mathbf{Z}_2$ $H_2(\mathbb{RP}^2; \widetilde{\mathbb{Z}}) = \mathbb{Z}$	$\widehat{D4}$ $\widehat{D0}$ D6 D2
5	$H_0(\mathbb{RP}^3; \mathbb{Z}) = \mathbb{Z}$ $H_1(\mathbb{RP}^3; \mathbb{Z}) = \mathbf{Z}_2$ $H_3(\mathbb{RP}^3; \mathbb{Z}) = \mathbb{Z}$	D5 D1 $\widehat{D4}$ $\widehat{D0}$ 2	$H_0(\mathbb{RP}^3; \widetilde{\mathbb{Z}}) = \mathbf{Z}_2$ $H_2(\mathbb{RP}^3; \widetilde{\mathbb{Z}}) = \mathbf{Z}_2$	$\widehat{D3}$ $\widehat{D(-1)}$ $\widehat{1}$ $\widehat{5}$
4	$H_0(\mathbb{RP}^4; \mathbb{Z}) = \mathbb{Z}$ $H_1(\mathbb{RP}^4; \mathbb{Z}) = \mathbf{Z}_2$ $H_3(\mathbb{RP}^4; \mathbb{Z}) = \mathbf{Z}_2$	D4 D0 $\widehat{D3}$ $\widehat{D(-1)}$ $\widehat{1}$	$H_0(\mathbb{RP}^4; \widetilde{\mathbb{Z}}) = \mathbf{Z}_2$ $H_2(\mathbb{RP}^4; \widetilde{\mathbb{Z}}) = \mathbf{Z}_2$ $H_4(\mathbb{RP}^4; \widetilde{\mathbb{Z}}) = \mathbb{Z}$	$\widehat{D2}$ $\widehat{4}$ $\widehat{0}$ 2
3	$H_0(\mathbb{RP}^5; \mathbb{Z}) = \mathbb{Z}$ $H_1(\mathbb{RP}^5; \mathbb{Z}) = \mathbf{Z}_2$ $H_3(\mathbb{RP}^5; \mathbb{Z}) = \mathbf{Z}_2$ $H_5(\mathbb{RP}^5; \mathbb{Z}) = \mathbb{Z}$	D3 $\widehat{D(-1)}$ $\widehat{D2}$ $\widehat{0}$ 2	$H_0(\mathbb{RP}^5; \widetilde{\mathbb{Z}}) = \mathbf{Z}_2$ $H_2(\mathbb{RP}^5; \widetilde{\mathbb{Z}}) = \mathbf{Z}_2$ $H_4(\mathbb{RP}^5; \widetilde{\mathbb{Z}}) = \mathbf{Z}_2$	$\widehat{D1}$ $\widehat{3}$ $\widehat{1}$
2	$H_0(\mathbb{RP}^6; \mathbb{Z}) = \mathbb{Z}$ $H_1(\mathbb{RP}^6; \mathbb{Z}) = \mathbf{Z}_2$ $H_3(\mathbb{RP}^6; \mathbb{Z}) = \mathbf{Z}_2$ $H_5(\mathbb{RP}^6; \mathbb{Z}) = \mathbf{Z}_2$	D2 $\widehat{D1}$ $\widehat{(-1)}$ $\widehat{1}$	$H_0(\mathbb{RP}^6; \widetilde{\mathbb{Z}}) = \mathbf{Z}_2$ $H_2(\mathbb{RP}^6; \widetilde{\mathbb{Z}}) = \mathbf{Z}_2$ $H_4(\mathbb{RP}^6; \widetilde{\mathbb{Z}}) = \mathbf{Z}_2$ $H_6(\mathbb{RP}^6; \widetilde{\mathbb{Z}}) = \mathbb{Z}$	$\widehat{D0}$ $\widehat{2}$ $\widehat{0}$ D2
1	$H_0(\mathbb{RP}^7; \mathbb{Z}) = \mathbb{Z}$ $H_1(\mathbb{RP}^7; \mathbb{Z}) = \mathbf{Z}_2$ $H_5(\mathbb{RP}^7; \mathbb{Z}) = \mathbf{Z}_2$	D1 $\widehat{D0}$ $\widehat{0}$	$H_0(\mathbb{RP}^7; \widetilde{\mathbb{Z}}) = \mathbf{Z}_2$ $H_2(\mathbb{RP}^7; \widetilde{\mathbb{Z}}) = \mathbf{Z}_2$ $H_4(\mathbb{RP}^7; \widetilde{\mathbb{Z}}) = \mathbf{Z}_2$ $H_6(\mathbb{RP}^7; \widetilde{\mathbb{Z}}) = \mathbf{Z}_2$	$\widehat{D(-1)}$ $\widehat{1}$ $\widehat{(-1)}$ $\widehat{1}$
0	$H_0(\mathbb{RP}^8; \mathbb{Z}) = \mathbb{Z}$ $H_1(\mathbb{RP}^8; \mathbb{Z}) = \mathbf{Z}_2$ $H_5(\mathbb{RP}^8; \mathbb{Z}) = \mathbf{Z}_2$	D0 $\widehat{D(-1)}$ $\widehat{(-1)}$	$H_2(\mathbb{RP}^8; \widetilde{\mathbb{Z}}) = \mathbf{Z}_2$ $H_6(\mathbb{RP}^8; \widetilde{\mathbb{Z}}) = \mathbf{Z}_2$	$\widehat{0}$ $\widehat{0}$

This in turn show us that we have actually two different cohomology groups for a \widehat{d} -brane given by H^{6-d} and H^{2-d} in the case $0 < p - d \leq 2$. However we also have a single cohomology group pointing out the presence of two different branes. H^{6-d} is related to \widehat{d} -branes as well as to $\widehat{(d \pm 4)}$ -branes, since $H^{6-d} = H^{2-(d-4)}$ (or vice versa, $H^{2-d} = H^{6-(d+4)}$).

In order to get a more exact picture, let us take advantage of our knowledge of K-theory. The T-dual versions of D-branes in Type I and Type $Usp(32)$ theories (recall, just those with no transversal coordinates to Op) are given as follows:

- In the presence of an Op^- -plane, we actually have Dp , $\widehat{D(p-1)}$, $\widehat{D(p-2)}$ and $D(p-4)$ -branes (since we are considering just orientifolds which dimension is less than seven).
- In the presence of an Op^+ -plane, we have \overline{Dp} , $D(p-4)$, $\widehat{D(p-5)}$ and $\widehat{D(p-6)}$ -branes.

Hence we conclude that some of the branes given in table 4 are in fact the D-branes contained in the above spectrum of branes. In particular,

- \widehat{d} -branes classified by H^{6-d} are in fact \widehat{Dd} -branes on top of an Op^- -plane.
- $\widehat{d-4}$ -branes classified by $H^{2-(d-4)}$ are in fact $\widehat{D(d-4)}$ -branes on top of an Op^+ -plane.

Finally, notice that topologically is allowed to relate d -branes with Op^+ -planes and $(d-4)$ -branes with Op^- -planes. We will discuss this possibility in the next section.

5.2 R-R and NS-NS fluxes

In order to prove that some of the branes obtained by wrapping higher or equal dimensional branes on suitable non-trivial homological cycles, are truly the T-dual version of the known D-branes classified by K-theory in Type I and $USp(32)$ string theories, we will use the topological relation between products of RR and NS-NS fluxes in Type II theories and D-branes, studied in [32].

Let us describe briefly the procedure which transforms topologically a non-BPS \widehat{Dd} -brane into a source-flux given by $H_{NS}G_{6-d}$ for Type II theories.

For Type II theories these couplings are given by

$$\int_{\mathcal{M}_{10}} H_{NS} G_{6-d} C_{d+1}, \quad (5.2)$$

with G_{6-d} being the RR field strength of C_{5-d} (with appropriate d for IIA or IIB theories). Topological couplings given by (5.2) show that there is the possibility to endow NS-NS and RR fluxes with charges under RR fields C_{d+1} , justly as Dd -branes. Thus, transitions between branes and configurations of suitable fluxes are possible. In [32] the case of the non-BPS $\widehat{D4}$ -brane was considered. Since these two systems are topologically equivalent, we are able to invert the procedure, i.e., having a source-flux of the form $H_{NS}G_{6-d}$, we can transform it into a Dd -brane of Type II theory.

For T-dual versions of branes on top of an $O9^+$ -plane, we consider the product of fluxes, far away from the orientifold, $H_{(7)}G_{2-d'}$, with $H_{(7)}$ being the magnetic dual of H_{NS} and $d' = d - 4$. This is because in the presence of an Op^+ -plane there is a magnetic NS-NS field in the background (remember that an Op^+ -plane is constructed by a NS5-brane intersecting an orientifold plane Op^-). The product $H_7G_{6-d'}$ can topologically be transformed into a $\widehat{Dd'}$ -brane.

By this procedure we are able to confirm our proposal concerning that cohomology groups (Poincaré dual of those given in table 4), are the relevant ones for Dd -branes on top of Op -planes.

For more details concerning the characteristics of these branes, see appendix A.

6 Physical Interpretation of RR Fields in K-theory

Up to here we have classified all RR-fields in a background given by Op^\pm -planes and d -branes by using K-theory. Also we have classified RR fields in the presence of d -branes on top of orientifold planes through cohomology. In this section we relate both descriptions by using the AHSS which in turn provides a physical interpretation of such d -dimensional subspaces when the fluxes are turned on.

6.1 d -branes as Dd -branes

The K-theory classification of RR fields given in tables 1 and 2 give us a lot of information. For instance, as it was said, for the case $d = p$ we get a truly classification of orientifold planes. In a different (but related) point of view, such a classification give us the possible p -branes present on top of Op -planes when discrete RR fluxes are turned on. By considering this latter alternative description, we can interpret the exotic orientifold planes \widetilde{Op}^\pm as composed by a “normal” Op -plane and a p -brane with certain RR charge (integer or discrete). However, for the case of $d < p$, the interpretation is not so obvious as the above one. Firstly, we notice that the K-theory classification of RR fields given in tables 1 and 2 is different from the RR charge K-theory classification. In

fact, the discrepancy is given by the second terms in the right-handed side of the fields in the above tables. Recall that these fields came from the second K-theory groups in the rhs in equations (4.21). On the other hand, the lhs terms in tables 1 and 2 which came from the first terms in the rhs of equations (4.21), actually give us the RR charges of *Dd-branes* on top of *Op*-planes. This can be easily inferred by noticing that such K-theory groups give the same result than the K-theory groups (classifying RR charges) given in eqs. (3.2) and (3.3).

Hence, in this case we can interpret physically the meaning of the d -dimensional submanifold related to the RR fields classified by K-theory. They are justly the *Dd*-branes with RR charge, i.e., the RR fields computed by $KR^{d-p}(\{pt\})$ for Op^- and $KH^{d-p}(\{pt\})$ for Op^+ (see equation (4.21)) have *Dd*-branes as sources. Then, our conclusion reads:

*d-branes associated to RR fields classified by K-theory through $KR^{d-p}(\{pt\})$ for Op^- and $KH^{d-p}(\{pt\})$ for Op^+ , are exactly the usual *Dd*-branes on top of orientifold planes. They are the T-dual version of the *D*-branes on top of $O9^-$ and $O9^+$ planes classified by Eqs. (3.2) and (3.3).¹²*

6.2 d -branes as d -fluxbranes?

Lets turn our attention to the RR fields not associated with a source. So, it is time to answer our third question raised in section 4.3. For that, let us start by giving a “cohomology” approach of the answer. We saw that RR fluxes G_{6-d} which are classified by the cohomology group H^{6-d} are related to truly \widehat{Dd} -branes on top of Op^- -planes. This conclusion was taken after using our K-theory knowledge of RR classification and by topologically transforming the product of fluxes $G_{6-d}H_{NS}$ into a \widehat{Dd} -brane.

As we said at the end of section 5, we can relate the fluxes G_{6-d} to $(d-4)$ -branes not in the presence of an Op^+ but in the presence of an Op^- instead. Clearly, this $(d-4)$ -brane can not be a Dirichlet brane, since we know by K-theory which branes are present in top of an orientifold plane. Hence, whatever these branes could be, they are associated to RR fields without source. Such RR fields are constructed far away from the orientifold plane, by the product of fluxes $G_{2-d'}H_{(7)}$, where as usual, $d' = d - 4$. Notice that in this situation, although we have an Op^- -plane, we take the product of the RR flux with the magnetic dual of H_{NS} . After all, this product is topologically

¹²As it was said, T-duality is taken on longitudinal coordinates on the *D*-branes on the ten-dimensional theories, Type I and Type $USp(32)$. When the number of compact coordinates is higher than the dimensionality of the *D*-brane, we get a brane which has some transversal coordinates to the orientifold plane (the T-dual version of $O9$). Such branes are not considered in this paper.

available. Physically can be understood as the presence of a NS5-brane far away from the orientifold.

On the other hand, RR fields that are not associated to any source (i.e., without any d' -dimensional objects charged under this field) can be only tangent to the d' -brane. This topological property allows to avoid sources for the fields. According to [18], this tangent field denoted as $F_{9-d'}$ must satisfy that

$$\int_{\partial\mathcal{M}_{9-d'}} F_{9-d'} < \infty, \quad (6.1)$$

with $\mathcal{M}_{9-d'}$ the $(9-d')$ -dimensional transverse space to a d' -dimensional object. If this field is extended over $\mathcal{M}_{9-d'}$ it does not require sources.

Now, in order to fix the notation, let us classify RR fields (by cohomology), with and without source, related to the same d -dimensional brane. Such RR fields (as we said, there are actually two cohomology groups classifying RR fields related to a d -brane) are classified by H^{6-d} and H^{2-d} . The latter one refers to RR fields that do not have a source in the presence of an Op^- -plane. The opposite situation holds for an Op^+ .

We argue that, this is the case for the RR fields given by the groups $KR^{d-10}(pt)$ and $KH^{d-10}(pt)$ in equation (4.21) (or, for the right handed fields in tables 1 and 2).

Hence, because they are source-free, they can be extended over \mathcal{M}_d and therefore,

$$\int_{\mathcal{M}_{9-d}} F_{9-d} < \infty. \quad (6.2)$$

This is precisely the property that a fluxbrane satisfies.

A flux d -brane (see [34, 35, 36, 37]), denoted as Fd -brane, is a $(d+1)$ -dimensional object with non-zero flux F_{9-d} on the $(9-d)$ -dimensional transverse space to the brane. This is contrasted with the usual Dd -branes which carry a RR charge measured by integrating out the field strength over a surrounding sphere. Also, fluxbranes are generalizations in higher dimensions of flux-tubes, that are solutions in General Relativity with precisely these properties. The most known example of it is the Melvin universe [38]. Basically this consists in a solution of the Einstein's equation for General Relativity in four dimensions, in where a 2-form field is present in the background and it is confined by its self-gravity.

We argue that the RR fields classified by $KR^{d-10}(\{pt\})$ for Op^- and $KH^{d-10}(\{pt\})$ for Op^+ (see equation (4.21)) are actually the field strength F_{9-d} related to fluxbranes. This is, the role of the d -dimensional subspaces for this kind of RR fields without source, is the presence of a flux d -brane Fd , or Fd -brane for short.

The interesting fact is that cohomology also captures the presence of the flux F_{9-d} by some unknown mechanism. Some of the objects classified by cohomology seem to be D-branes at that level, but in K-theory are related to source-free RR fields. A more deeper study of this features is required but it is beyond the scope of this paper.

From now on, we will denote the d -dimensional objects related to RR fields without source as “F d ”-branes which has related a \mathbf{Z}_2 field. This notation remarks our limited knowledge about their nature.

6.3 Example. Branes and Fluxes in the $O5$ -plane

We are ready to apply all the information we have got in the previous sections. On one hand we have the cohomological classification of RR fields in the presence of Op -planes and d -branes. Also, we were able to infer some of the properties of such branes and the role they are playing on. The same was done in the case of the K-theory classification of RR fields given in section 4. The final step is to relate both of them by the AHSS as was done in Ref. [5] by BGS.

Let us do it by analyzing a concrete example: the orientifold five-plane. We will analyze the case for each value of d in the presence of an $O5$ -plane. The case $d = 5$ has already been studied in previous sections, although there is some extra information which is important to point out.

Five brane

According to our discussion at the beginning of this section, it is possible to describe the exotic orientifold five-planes as:

$$\begin{aligned}\widetilde{O5}^- &= O5^- + \tfrac{1}{2}D5, \\ \widetilde{O5}^+ &= O5^+ + \text{“F5”}.\end{aligned}\tag{6.3}$$

We do not know exactly what “F5” could be, but as it is classified by the second term in the left hand side of (4.21), which corresponds to a RR field without a source. We argue that this is a fluxbrane F5 with \mathbf{Z}_2 charge and moreover, obeys a T-duality relation given by Eq. (B.10) (see appendix B), at the cohomology level.

Four-brane

According to table 4, the (co)homology group for a 4-brane on an $O5$ -plane is given by

$$H_1(\mathbb{RP}^3, \mathbb{Z}) \cong H^2(\mathbb{RP}^3, \mathbb{Z}) = \mathbf{Z}_2.\tag{6.4}$$

The K-theory groups are

$$\begin{aligned} O5^- : \quad KR^{-6}(\mathbb{S}^{4,0}) &= \mathbf{Z}_2, \\ O5^+ : \quad KH^{-6}(\mathbb{S}^{4,0}) &= \mathbf{Z}_2. \end{aligned} \tag{6.5}$$

Now, we can proceed to build the corresponding sequence in order to resolve the extension problem addressed by the AHSS. d_3 is also trivial for both cases, and we find that

$$\begin{aligned} K_0 = K_1 = K_2 &= \mathbf{Z}_2 \quad \text{for both cases} \\ K_2/K_3 &= H^2 = \mathbf{Z}_2 \\ K_3 &= 0. \end{aligned} \tag{6.6}$$

The extension problem is given by the exact sequence

$$\begin{array}{ccccccc} 0 & \longrightarrow & K_3 & \longrightarrow & K_2 & \longrightarrow & K_2/K_3 \longrightarrow 0 \\ & & \parallel & & \parallel & & \parallel \\ & & 0 & & \mathbf{Z}_2 & & \mathbf{Z}_2 \end{array} \tag{6.7}$$

This is trivial and it is concluded that there are not effects on both $O5^\pm$ -planes, due to the torsion flux G_2 , i.e, cohomology and K-theory descriptions coincide. For the $O5^-$ -plane, this is the T-dual version of the $\widehat{D8}$ -brane in Type I theory, while for the $O5^+$ -plane, the presence of a topological 4-dimensional object is unexpected. As for the five branes, we can interpret this brane as the result of turning on a discrete RR field (without sources) over a 4-dimensional submanifold of the orientifold five-plane. We argue that this is related to a 4-fluxbrane (or a “F4”-brane). It would be very interesting the study of anomalies in these objects and their relation to anomalies of fluxes described in [32]. According to equation (A.2) in the appendix A, this 4-brane is T-dual related to a 4-brane on an $O4^+$ - and $O6^+$ -planes; this is obtained by the Eq. (B.10) at the cohomology level.

Three-brane

The cohomology group which classifies three branes on top of $O5$ -planes is $H^3(\mathbb{RP}^3, \mathbb{Z})$, and the K-theory groups are given by

$$\begin{aligned} O5^- : \quad KR^{-7}(\mathbb{S}^{4,0}) &= \mathbf{Z}_2, \\ O5^+ : \quad KH^{-7}(\mathbb{S}^{4,0}) &= 0. \end{aligned} \tag{6.8}$$

In the case of an $O5^+$ -plane, the map $d_3 : H^0(\mathbb{RP}^3) \rightarrow \widetilde{H}^3(\mathbb{RP}^3)$ is surjective; this means that the flux G_3 is lifted to a trivial class in K-theory. Physically this means

that there are not any type of three-branes on top of an $O5^+$ -plane (neither D-branes nor “fluxbranes”). For the $O5^-$, d_3 is trivial and the extension problem is given by

$$\begin{array}{ccccccc}
0 & \longrightarrow & K_4 & \xrightarrow{id} & K_3 & \xrightarrow{id} & K_3/K_4 \longrightarrow 0 \\
& & \parallel & & \parallel & & \parallel \\
& & 0 & & KR^{-7}(\mathbb{S}^{4,0}) = \mathbf{Z}_2 & & H^3(\mathbb{RP}^3; \widetilde{\mathbb{Z}}) = \mathbf{Z}_2
\end{array} \tag{6.9}$$

The extension is trivial and we conclude that this brane is the T-dual version of the $\widehat{D7}$ -brane in Type I theory.

Two-brane

Possible two-branes are obtained by wrapping a D5-brane on the non trivial untwisted and compact 3-cycle of \mathbb{RP}^3 . The 3-cycle is classified by the untwisted homology group

$$H_3(\mathbb{RP}^3, \mathbb{Z}) \cong H^0(\mathbb{RP}^3, \mathbb{Z}) = \mathbb{Z}. \tag{6.10}$$

However this integral flux has another interesting interpretation. As was pointed out in [5, 6], this flux is related to massive IIA supergravity [39, 40].

In order to look for some correlations, we need to solve the extension problem given by the AHSS. In the case of $O5^-$ this reads,

$$\begin{array}{ccccccc}
0 & \longrightarrow & K_1 & \longrightarrow & K_0 & \xrightarrow{id} & K_0/K_1 \longrightarrow 0 \\
& & \parallel & & \parallel & & \parallel \\
& & 0 & & KR^{-8}(\mathbb{S}^{4,0}) = \mathbb{Z} & & H^0(\mathbb{RP}^3; \mathbb{Z}) = \mathbb{Z}
\end{array} . \tag{6.11}$$

This is trivial and admits just one solution (the trivial one). The integer flux described by K-theory indicates the presence of massive D2-branes [41, 42]. Moreover, for the $O5^+$ -plane there is a surjective map $d_3 : H^0 = \mathbb{Z} \rightarrow \widetilde{H}^3 = \mathbf{Z}_2$ which implies that odd values of G_0 are not allowed. This must be related to an anomaly in the three-dimensional gauge theory on 2-branes on top of an $O5^+$ -plane with odd G_0 . These two-branes could be related to two-fluxbranes. It would be very interesting to study these systems and their possible anomalies.

One-brane

Essentially we have the same cohomology and K-theory groups as for the five-branes on both kind of orientifolds. However the difference is that the K-theory groups are

inverted respect to the five-branes. We are not describing our calculations in detail but just focusing in the results and in their physical interpretation.

For the $O5^-$ -plane we have a D1-brane (the usual one) carrying an integer RR charge. Also we have an induced “F1”-brane. For the $O5^+$ -plane we have also the usual D1-brane expected by T-duality, that corresponds to the D5-brane on Type $USp(32)$ string theory, and a fractional integer one-brane, $\frac{1}{2}$ D1-brane.

Zero-brane

In this case we have the same situation as in the case for the 4-branes. The result is that for the $O5^-$ -plane we have an induced “F0”-brane with topological charge \mathbf{Z}_2 . For the $O5^+$ -plane we have the expected $\widehat{D0}$ -brane. “F0”-brane obeys a T-duality relation given by Eq. (B.10).

(-1)-brane

The case of the (-1)-brane is very interesting and we analyze it in more detail. According to table 4, the cohomology group which classifies RR fields related to (-1) -branes is $H^3(\mathbb{RP}^3, \widetilde{\mathbb{Z}}) = \mathbf{Z}_2$.

For the $O5^+$ -plane, there exists a surjective map

$$d_3 : H^0(\mathbb{RP}^3, \mathbb{Z}) = \mathbb{Z} \rightarrow H^3(\mathbb{RP}^3, \widetilde{\mathbb{Z}}) = \mathbf{Z}_2 \quad (6.12)$$

and the flux G_3 is lifted to a trivial element in K-theory. This means that a (-1) -brane is unstable and decay to vacuum when an $O5^+$ is present. However, in order to get a better picture of this situation, we must resolve the extension problem addressed by the AHSS. We found that $K_3 = KH^{-11}(\mathbb{S}^{4,0}) = \mathbf{Z}_2$ and that $K_4 = K_3/K_4 = 0$. Hence, the sequence would be trivial only if the K-theory class was zero. Physically this means that although K-theory actually classifies the RR charge of (-1) -branes, it also establishes an extra condition: only an even number of (-1) -branes is allowed to be on top of an $O5^+$ -plane, i.e., (since the RR charge of such branes is \mathbf{Z}_2) the K-theory charge must be cancelled.

This resembles the behavior of the $\widehat{D3}$ -brane in $USp(32)$ theory where a single three-brane is unstable, but it cannot decay to the vacuum because it has a discrete \mathbf{Z}_2 charge. Thus, it is expected that K-theory measures this charge, but it does not allow the presence of a single non-BPS D-brane. This is actually the required topological condition on the $\widehat{D3}$ -brane on top of an $O9^+$ -plane placed at a point in \mathbf{T}^6 , described in section 3, in order to cancel global gauge anomalies on suitable probe branes. Here we have the same condition applied to a T-dual version of such a system (notice that by

considering a T-dual version of $USp(32)$ string theory, we are actually compactifying the system on a torus \mathbf{T}^4 where the (-1) -brane is placed at a point). As was shown in Refs. [31, 43], this is also a property of the $\widehat{D7}$ -brane in Type I theory.

The case of the Op^- is puzzled. We obtain a non-zero value by cohomology but it is zero by K-theory. As K-theory is given exactly by the graded complex and then by cohomology, this is somewhat contradictory. We do not know how to explain this feature, although we think that a more deeper study on differences at the cohomology level for branes on top of Op^- or Op^+ , could be very helpful in order to explain the above puzzle. Notice however that (-1) -branes given by cohomology actually reproduce the expected (-1) -branes classified by K-theory.

Finally, for all different values of d and p , we resume our results in table 5.

6.4 The Case for $p \leq 2$

It was shown in [4, 5] that for $p \leq 2$ there are some extra interesting features for both orientifolds Op^\pm . In this case there are additional RR discrete fluxes classified by the cohomology group H^{2-p} . Let us summarize some results given in [5]:

- In the case of $O2^-$ we have actually three fluxes to be considered. G_6 stands for the presence of a BPS D2-brane on top of the orientifold. G_4 is the one related to the exotic plane $\widetilde{O2}^-$ and finally G_0 is the one related to $\widehat{O2}$. The last one is interpreted as a massive D2-brane considered previously.
- In the case of $O2^{+-}$, G_0 is twice an integer. This means that there is not allowed \mathbf{Z}_2 -fluxes in K-theory, and a $O2^+$ is equivalent to an $\widetilde{O2}^+$ -plane, but massive D2-branes are still present besides the usual BPS D2-branes.
- For $O1^-$ and $O0^-$ ($\widetilde{O1}^-$ - and $\widetilde{O0}^-$ -planes) we have the usual D1 and D0-branes (respectively) and the induced fractional $\frac{1}{2}D1$ and $\frac{1}{2}D0$. Hence, we can write,

$$\begin{aligned}\widetilde{O1}^- &= O1^- + \frac{1}{2}D1, \\ \widetilde{O0}^- &= O0^- + \frac{1}{2}D0.\end{aligned}\tag{6.13}$$

- For $O1^+$ and $O0^+$ we also have equivalent orientifolds due to the fact that there are surjective maps

$$\begin{aligned}d_3 : H^2 = \mathbf{Z}_2 &\rightarrow \widetilde{H}^5 = \mathbf{Z}_2, & \text{for } O1^+ \\ d_3 : H^3 = \mathbf{Z}_2 &\rightarrow \widetilde{H}^6 = \mathbf{Z}_2, & \text{for } O0^+.\end{aligned}\tag{6.14}$$

Table 5: Brane states on top of Op^- -planes considering discrete RR fields. Left superscript m stands for the massive D2-branes. In the case of D6, this is fractional and for the $O6^+$ the flux is twice an integer [5]. For the Op^+ -planes, D p -branes are actually anti-D p -branes by tadpole cancellation in $USp(32)$ string theory.

$O6^-$	$O5^-$	$O4^-$	$O3^-$
D6+ ^{m} D6			
$\widehat{D5}$	D5+ $\frac{1}{2}$ D5		
$\widehat{D4}$	$\widehat{D4}$	D4+ $\frac{1}{2}$ D4	
-	$\widehat{D3}$	$\widehat{D3}$	D3+ $\frac{1}{2}$ D3
D2+ ^{m} D2	^{m} D2	$\widehat{D2} + ^mD2$	$\widehat{D2} + ^mD2$
“F1”	D1+“F1”	“F1”	$\widehat{D1} + \text{“F1”}$
“F0”	“F0”	“F0” + D0	“F0”
$O6^+$	$O5^+$	$O4^+$	$O3^+$
D6+ ^{m} D6			
“F5”	D5+“F5”		
“F4”	“F4”	D4+“F4”	
-	-	-	D3
D2+ ^{m} D2	^{m} D2	^{m} D2	^{m} D2
$\widehat{D1}$	D1+ $\frac{1}{2}$ D1	-	-
$\widehat{D0}$	$\widehat{D0}$	D0 + $\frac{1}{2}$ D0	-
-	$\widehat{D(-1)}$	$\widehat{D(-1)}$	D(-1)

Then, $O1^+$ and $\widetilde{O1}^+$ -planes are equivalent in K-theory. The same equivalence is found for $O0^+$ and $\widetilde{O0}^+$ -planes.

By comparing these results given by K-theory with the cohomology classification of branes (which was discussed in section 5), we conclude:

- $\widetilde{Op}^- = Op^- + \frac{1}{2}Dp$ for all p . It is represented by the integer flux \mathbb{Z} in K-theory. (Of course, this is obtained by the use of the AHSS).
- $\widehat{Op}^- = Op^- + \text{“Fp”}$ for $p < 2$. “Fp” is represented by the K-theory flux \mathbf{Z}_2 .
- $\widehat{\widetilde{Op}}^- = Op^- + \frac{1}{2}Dp + \text{“Fp”}$. Hence, the K-theory flux $\mathbb{Z} \oplus \mathbf{Z}_2$ represents the existence of these type of orientifold planes for $p < 2$. For $p > 2$ we only have a \mathbb{Z} charge and this means that there is only one possibility of constructing an exotic orientifold: the \widetilde{Op}^- -plane.

For the Op^+ -plane, we have:

- $\widetilde{Op}^+ = Op^+ + \text{“Fp”}$ for $6 > p > 3$. With “Fp” being the brane obtained at cohomology level by the normal or twisted group, H^{6-p} . This is represented in K-theory by the flux $\mathbb{Z} \oplus \mathbf{Z}_2$.
- $\widetilde{Op}^+ = Op^+$ for $p \leq 3$. Although there is a cohomology group related to a p -brane, this is lifted to a zero class in K-theory through the differential map $d_3 : H^{2-p} \rightarrow H^{5-p}$. Then, the possible “Fp”-brane is classified in K-theory by the zero class. These orientifolds are equivalent.
- $\widehat{Op}^+ = Op^+ + \text{“Fp”}$ does not exist for $p < 2$. This is because the cohomology class G_{2-p} related to the p -brane is obstructed to be lifted to K-theory (again, by the presence of the non-trivial map d_3). Hence, “Fp”-brane, related to H^{2-p} is not classified by K-theory. In this sense, it is physically absent.

Now, by applying the AHSS to the case for d -branes on top of Op -planes with $p \leq 2$, we get the results given in table 6.

6.4.1 Equivalent and Unexistent branes

From the point of view of cohomology, since we have two different cohomology groups associated to the same kind of branes, we can construct two different kind of orientifold planes. The existence of this planes depends if the relevant fluxes can be lifted to non-trivial classes in K-theory. For Op^- we have that always, the cohomology groups

Table 6: Brane states on top of Op -planes considering discrete RR fields, $p \leq 2$.

$O2^-$	$O1^-$	$O0^-$
$D2 + {}^m D2 + \frac{1}{2} D2$	-	-
$\widehat{D1} + \text{"F1"}$	$D1 + \text{"F1"} + \frac{1}{2} D1$	-
$\widehat{D0} + \text{"F0"}$	$\widehat{D0} + \text{"F0"}$	$D0 + \text{"F0"} + \frac{1}{2} D0$
-	$\widehat{D(-1)}$	$\widehat{D(-1)}$
$O2^+$	$O1^+$	$O0^+$
$D2 + {}^m D2$	$D1$	$D0$

related to d -branes can be lifted to non-trivial elements in K-theory, and that is why we have three different kinds of Op^- -planes.

For the Op^+ -plane the situation is different since there exists a non-trivial map d_3 which obstructs any lifting of G_{2-p} fluxes to K-theory. This is the reason that $\widehat{O1}^+, \widehat{O0}^+, \widetilde{\widehat{O1}}^+$ and $\widetilde{\widehat{O0}}^+$ do not exist. By extending this argument to all the possible d -branes on top of orientifold planes, we have that the following branes do not exist:

$$\begin{aligned}
O5^+ : & \quad D2 \\
O4^+ : & \quad D2, \widehat{D1} \\
O3^+ : & \quad D2, \widehat{D1}, \widehat{D0},
\end{aligned} \tag{6.15}$$

and the following ones are represented by zero class in K-theory,

$$\begin{aligned}
O5^+ : & \quad \widehat{D3}, \widehat{D(-1)} \\
O4^+ : & \quad \widehat{D3}, \widehat{D1}, \widehat{D(-1)} \\
O3^+ : & \quad \widehat{D3}, \widehat{D2}, \widehat{D1}.
\end{aligned} \tag{6.16}$$

The same effect is observed for d -branes on top of Op^+ -planes. Consider for instance the 1-brane on top of an $O2^+$ -plane. From cohomology there are two sources for possible $\widehat{1}$ -branes. However, one of them, the G_5 -flux is lifted to a trivial element in K-theory, through the surjective map $d_3 : H^2 = \mathbf{Z}_2 \rightarrow \widetilde{H}^5 = \mathbf{Z}_2$. The other one, the $G_1 \in \widetilde{H}^1 = \mathbf{Z}_2$ flux is obstructed to be lifted to K-theory because the map $d_3 : \widetilde{H}^1 \rightarrow H^4$. Then there are no possible RR fluxes captured by K-theory for the $O2^+$ -plane. The same happen with d -branes with $d < p$ and for $p \leq 2$ for the Op^+ -plane. This is shown in table 7.

The conclusion is that we can deduce the existence (or not existence) of certain D-branes by computing first their cohomology group and then lifting their classes to K-theory by the d_3 differential map. In find that some branes do not exist even though cohomology suggests their existence.

Table 7: Equivalent-vacuum and obstructed branes in K-theory on top of Op^+ -planes

Orientifold	d_3 map	Branes \cong vacuum	Unexistent branes
O5	$H^0 \rightarrow \widetilde{H}^3$	$\widehat{D3} \quad \widehat{D(-1)}$	D2
O4	$\widetilde{H} \rightarrow H^4$ $H^0 \rightarrow H^3$	$\widehat{D2}$ $\widehat{D3} \quad \widehat{D(-1)}$	$\widehat{D1}$ D2
03	$H^2 \rightarrow \widetilde{H}^5$ $H^0 \rightarrow \widetilde{H}^3$ $\widetilde{H}^1 \rightarrow H^4$	$\widehat{D1}$ $\widehat{D3}$ $\widehat{D0}$	D2
O2	$\widetilde{H}^3 \rightarrow H^6$ $\widetilde{H}^1 \rightarrow H^4$ $H^2 \rightarrow \widetilde{H}^5$ $H^0 \rightarrow \widetilde{H}^3$	$\widehat{D0}$ $\widehat{D2}$ $\widehat{D1}$ $\widehat{D(-1)}$	$\widehat{D(-1)}$ $\widehat{D1}$ $\widehat{D0}$
O1	$H^2 \rightarrow \widetilde{H}^5$ $H^0 \rightarrow \widetilde{H}^3$ $\widetilde{H}^1 \rightarrow H^4$ $\widetilde{H}^3 \rightarrow H^6$	$\widehat{D1}$ $\widehat{D(-1)}$ $\widehat{D0}$	$\widehat{D0}$ $\widehat{D1}$ $\widehat{D(-1)}$
O0	$H^4 \rightarrow \widetilde{H}^7$ $H^2 \rightarrow \widetilde{H}^5$ $H^0 \rightarrow \widetilde{H}^3$ $\widetilde{H}^3 \rightarrow H^6$	$\widehat{D(-1)}$ $\widehat{D(-1)}$ $\widehat{D0}$	$\widehat{D0}$ $\widehat{D(-1)}$

Furthermore, with the help of the AHSS we can explain the differences between cohomology and K-theory. For instance, the possible $\widehat{D2}$ -branes in table 4 do not exist for $O4^+$ -plane. This is because this brane is equivalent to vacuum in K-theory. The $\widehat{D1}$ brane does not exist for $O4^+$ because its respective flux is obstructed. We show in table 7, all the states that are equivalent to vacuum for $d < p$. For $d = p$ it was obtained that Op^+ is equivalent to \widetilde{Op}^+ . Also we show obstructed states on Op^+ and their corresponding maps.

Some important remarks are in order: firstly, there are two types of vacuum-equivalent branes. These are branes that have zero value in K-theory, as the $\widehat{D3}$ -brane on the $O5^+$ -plane. They do not exist because by K-theory their charge must be zero (added to the fact that non-zero cohomological value is lifted to a zero one in K-theory). The other type is a brane that its cohomological flux-value is lifted to a zero one in K-theory but is not zero measured by K-theory groups. This give us a topological condition (by the AHSS) about its charge. The main examples of this

type of branes are the $\widehat{D(-1)}$ -brane on $O4^+$ – and $O5^+$ –planes. We interpret this fact as the condition that discrete charge must be canceled on compact spaces. Thus, we see that by understanding the relation between cohomology and K-theory, we can give a picture about what it is the reason that global gauge anomalies, on suitable probe branes, should be canceled. This is the same global gauge anomaly computed at the end of section 3.

The absence of certain branes is explained just by obstruction in lifting cohomology classes to K-theory. There are some branes that are obstructed and equivalent to vacuum. For them, also a K-theory computation gives a zero flux-value.

7 Conclusions

In this paper we have classified RR fields by K-theory, in string backgrounds including orientifold planes Op^\pm and d -branes on top of them. We consider only d -branes with all their coordinates being longitudinal to the orientifold plane. In the case $d = p$ we actually recover the orientifold classification given in Ref. [5]. Some of these branes turn out to be actually Dd -branes (sources of the RR fields classified by K-theory), but also we find that some of such RR fields are not in fact related to a source. So, the nature of these branes is not totally clear, although we give some arguments which allow us to think that these branes are related to the well-know fluxbranes. Our notations of these branes, “ Fd ”, stands for our limited knowledge of their nature.

On the other hand, in order to get information about the general case $d < p$, we need (in the spirit of [5]) a cohomology classification of RR fields in such backgrounds and the use of the AHSS. By wrapping $D(d+n)$ -branes on compact non-trivial homological n -cycles of the transverse space of the d -brane, \mathbb{RP}^{8-p} we find the cohomology groups classifying RR fields in these systems. Many new results are found when we apply the AHSS to the above both classifications. For instance, we find that besides the expected D-branes on top of orientifold planes, actually there are more branes related to discrete RR fluxes. Some of them turn to be fractional Dd -branes and the other ones “ Fd ”-branes. In fact, in the case $d = p$, the presence of these extra branes give us the two exotic types of orientifold planes that we already knew: \widetilde{Op}^- and \widehat{Op}^- (for $p \leq 2$).

We also show that by analyzing all possible differential d_3 mappings, we were able to explain the reason why some d -branes (Dd -branes and “ Fd ”-branes) do not exist for certain values of d on top of an Op –plane. Indeed, for the case $d = p$ this fact reproduce one result given in [5]: the absence of certain exotic orientifold planes, labeled as \widehat{Op}^+

and $\widehat{Op^+}$ for $p < 2$.

Interesting enough, we also find that in the presence of an $O5^+$ - the $\widehat{D(-1)}$ -brane has to appear in an even number of them, in order we have a total zero topological charge (the topological charge of $\widehat{D(-1)}$ is \mathbf{Z}_2). Then, since this is the condition to cancel discrete charges on compact spaces and to avoid global gauge anomalies on suitable probe branes, we conclude that this is an effect of going from cohomology to K-theory. This is the same condition the $\widehat{D3}$ - and $\widehat{D4}$ -branes must satisfy on presence of an $O9^+$ -plane when they are placed at a point on compact spaces.

Finally we could explain (see appendix) why the “ Fd ”-brane seems at first sight to violate T-dual relations. This is because we have to apply T-duality on the $D(d+n)$ -branes wrapped on non-trivial homological cycles. Studying the procedure carefully we can conclude that “ Fd ”-branes also satisfy T-duality. One would wonder if these “ Fd ”-branes have some relation to the stable non-BPS states found in Refs. [44, 45].

It will be interesting to study the M-theory lifting of the states described in this paper and observe how the correlations and obstructions given by the differential maps and the AHSS are manifested in M-theory.

Also, it would be interesting to study a more general cycle in which we wrap 8-branes in order to pick up RR fields for d -branes on top of Op -planes. This requires to compute more general K-theory groups as $KR^n(\mathbb{S}^{l,m})$. These kind of cycles could give rise to a more interesting non-trivial effects, because the 8-branes could be wrapped into a “mixture” of the cycles considered in (4.13).

In Refs. [46, 47] it was found the correct twisted equivariant real K-theory which classify all the brane spectrum for certain orientifold models. In our paper it was not considered many states included in those models and it would be very interesting to find a relation of our results with those of Ref. [47], by wrapping $D(d+n)$ -branes on n -cycles, but taking all possible values of d (i.e., $d > p$) and by finding homology groups for more general orientifolds.

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A Topological transformation of R-R and NS-NS fluxes

In this appendix we study in detail how some of the d -branes obtained by wrapping D-branes on homology cycles are actually truly D-branes on top of Op -planes. We use the topological transformation mentioned in section 5.2 and some of the properties we already know for D-branes classified by K-theory. We analyze the following interesting cases:

- For $\widehat{D(p-1)}$ on an Op^- -plane and $\widehat{D(p-5)}$ on an Op^+ -plane¹³, the relevant cohomology group is

$$H^{7-p}(\mathbb{RP}^{8-p}) = \mathbf{Z}_2. \quad (\text{A.1})$$

The RR flux near to the orientifold plane is $G_{7-p} \in \mathbf{Z}_2$. Now, consider a local region far away from the orientifold plane. In such a region, the local theory is the Type II theory. There, H_{NS} and G_{7-p} are trivial forms (in the cohomology sense) and coincide with those of Type II theories. For Type IIA(B) theory, p is even(odd) and then the RR flux G_{7-p} does not exist (in both cases). This means that the $\widehat{D(p-1)}$ brane on top of an Op^- -plane cannot be separated from the orientifold plane, because far away from the orientifold, the RR flux becomes unstable since it is the RR flux associated to an unstable non-BPS D-brane of Type IIB theory. This RR flux eventually become stable just after the orientifold projects out the associate tachyon [33]. The RR field C_{p-1} is zero and does not couple to any D-brane (or in other words, this brane has zero RR charge). This is precisely the T-dual version of the behavior of the non-BPS $\widehat{D8}$ -brane in Type I theory. So, by this procedure, we confirm that relating the cohomology group $H^{7-p}(\mathbb{RP}^{8-p})$ to $\widehat{D(p-1)}$ gives also the expected behavior of a T-dual version of the $\widehat{D8}$ -brane in Type I theory. The same happens for the T-dual version of the $\widehat{D4}$ -brane in $USp(32)$ string theory using the magnetic dual NS-NS field $H_{(7)}$.

- For the $\widehat{D(p-2)}$ -brane on top of an Op^- -plane and for the $\widehat{D(p-6)}$ -brane on top of an Op^+ -plane, we found that the relevant cohomology group is

$$H^{8-p}(\mathbb{RP}^{8-p}) = \mathbf{Z}_2, \quad (\text{A.2})$$

¹³Although we do not know yet which branes are related to the different kind of orientifold planes, we infer that they are T-dual versions of D-branes on top of an $O9^-$ and $O9^+$ planes. This will be confirmed by the use of K-theory and the AHSS.

where this group is twisted if the cohomology group classifying BPS Dp -branes is untwisted and vice versa. Far away from the Op^- -plane we can build the flux $H_{NS}G_{8-p}$ which couples to the RR field C_{p-1} in the form

$$\int_{M_{10}} H_{NS}G_{8-p}C_{p-1}. \quad (\text{A.3})$$

For p being an even number (i.e. Type IIA theory far away from the orientifold plane), the RR flux G_{8-p} is an even rank form and it does exist. The same is true for p odd. Then, we are able to separate the $\widehat{D(p-2)}$ -brane from the orientifold by transforming the product of fluxes into branes. The flux $H_{NS}G_{8-p}$ is odd under the orientifold projection, because according to the relation (2.7), G_{8-p} is even and H_{NS} is odd. This means that in both sides of the orientifold we have transformed topologically the product of fluxes into a $D(p-2)$ -brane and a $\overline{D(p-2)}$ -brane. They carry opposite charge by the above argument or by the fact that the RR field C_{p-1} (which couples the $(p-2)$ -branes) is odd under the orientifold projection.

This corresponds to the T-dual version of the $\widehat{D7}$ -brane on Type I theory. The $\widehat{D7}$ -brane can be constructed, as a pair of $D7+\overline{D7}$ modulo the orientifold action. In other words, the D-seven-brane in Type IIB theory is unstable due to the tachyon in its spectrum, but stable when it is on top of the $O9^-$ -plane (the tachyon mode is removed out by the orientifold action).

Notice, that the flux G_{8-p} has non-trivial discrete values when it is near from the orientifold plane, but has trivial cohomology values when it is far away from the orientifold plane. This reflects the fact that just on the orientifold plane, we have stable ‘non-BPS’ branes, but far away from it, we are able to decompose the brane into stable or unstable D-branes in Type II theories. The same happens for the $\widehat{D(p-6)}$ -brane on top of an Op^+ -plane as a T-dual version of the non-BPS $\widehat{D3}$ -brane in $USp(n)$ string theory.

This procedure confirms again that the cohomology groups associated to the D-branes give all the expected properties of the known branes (T-dual versions of Type I and $USp(32)$ string theories).

- For the \widehat{p} -branes, the relevant cohomology group found was

$$H^{6-p}(\mathbb{RP}^{8-p}) = \mathbf{Z}_2, \quad (\text{A.4})$$

(twisted or untwisted). The flux G_{6-p} near to the orientifold plane (positive or negative type) has a non-trivial discrete value. However, far from it, it has a

trivial cohomology value corresponding to a RR flux in Type II theories. If p is odd, we have the Type IIB theory, and for p even we have the Type IIA theory. Then, a product of fluxes can be built in the bulk, as $H_{NS}G_{6-p}$ which is even under the orientifold projection because both H_{NS} and G_{6-p} are odd, or because the RR field coupling this product of fluxes C_{p+1} is even under the orientifold projection. This means that by using the flux G_{6-p} we are able to construct a product of fluxes which can be transformed topologically into Dp -branes in the bulk. This is consistent with the possibility of separating a Dp -brane from the orientifold. However, this kind of branes acquire a non-trivial discrete RR charge when they are on top of the orientifold plane. We know by Ref. [5] that in the case of an Op^- -plane this implies that we have a fractional Dp -brane, but the question remains open for the Op^+ -plane until the use of K-theory. We study this issue in the section 6.

B T-duality Relations

The RR fields not related to sources and listed by the right hand terms in tables 1 and 2 are given by the K-theory group $KR^{d-10}(\{pt\})$ and then at first sight, it seems that these fields do not obey T-duality rules, but they actually do. Looking at table 4 we can relate them to some of the D-branes provided by cohomology. In this appendix we show how T-duality applied to “the cohomology construction” explains the apparent T-duality violation and in the process we also report some interesting relations at the cohomological level. Nevertheless, it is required further analysis in order to obtain a realistic physical interpretation.

B.1 Distinguishing D6 and fractional D6-branes on an $O6^-$ -plane from cohomology

We found a ‘puzzle’ when we consider two D6-branes on top of an $O6^-$ -plane. If these integer-charged branes are obtained by wrapping D8- and D6-branes on 2- and 0-cycles of \mathbb{RP}^2 respectively, how can we distinguish which one is the integer charged D6-brane and which one is the half-integer brane predicted by K-theory correlations as shown in [5]?

We can resolve the apparent puzzle by using T-duality. Take a D8 brane expanded along coordinates 012345678 on an $O6^-$ -plane on 0123456 coordinates. We can wrap coordinates 78 on a 2-cycle of \mathbb{RP}^2 and obtain a D6-brane. Also we can take a D6-brane

on 0123456 coordinates and wrap it on a 0-cycle of \mathbb{RP}^2 and obtain a D6-brane.

With the T-duality relations we are able to elucidate which of them is fractional. Take T-duality on the 6 coordinate. This yields:

- Two $O5^-$ -planes on 012345
- A D7 brane on 01234578 that is wrapped on a 2-cycle of \mathbb{RP}^3 (a 2-cycle on \mathbb{RP}^3 is transverse to $O5^-$, on 78 coordinates after \mathbf{Z}_2 projection). This gives a $\widehat{D5}$ -brane, i.e., this is the G_1 -flux that by the AHSS induces a half-integer shift on the G_3 flux that corresponds to a half D5-brane [5]. Then, we found that this is precisely the D8-brane wrapped on a 2-cycle of \mathbb{RP}^2 which gives the fractional D6-brane.
- A D5 brane on 012345 that is wrapped on a 0-cycle. This gives the usual D5-brane on top of an $O5^-$ -plane.

The second point is confirmed also by T-dual processes depicted in [6]. If we want to build an $\widetilde{O7^-}$ -brane by a T-dual transformation on the system $O6^- + \widetilde{O6^-}$, we have to divide the two objects by an odd number of D8-branes as domain walls. But wrapping a D8 on a $\widetilde{O6^-}$ gives a half-integer shift on RR charge. Then when D8 branes shrinks to a point, that precisely is possible by the non-trivial 2-cycle on \mathbb{RP}^2 , a pair of $O6^-$ and $\widetilde{O6^-}$ -planes reduces to a pair of $O6^-$ -planes. Then T-dual configuration is always an $O7^-$ -plane.

In other words, taking T-duality on coordinate 7, we get:

- (By two $O6^-$ -planes) An $O7^-$ -plane.
- A D7 brane on 01234568 wrapped on a 1-cycle gives a 6-brane. However this 6-brane is T-dual to the fractional D6 on an $O6^-$ -plane. Considering such brane, implies that we have an $\widetilde{O6^-} = O6^- + \frac{1}{2}D6$. But we know from Ref. [6] that this system reduces to just $O6^-$ -planes. The absence of a D6-brane on an $O7^-$ -plane confirms that a D8-brane wrapped on a 2-cycle of \mathbb{RP}^2 (and classified in cohomology by $H^0(\mathbb{RP}^2, \mathbb{Z}) = \mathbb{Z}$) corresponds to the fractional D6-brane.
- A D7 brane on 01234567 wrapped on a 0-cycle. This is the usual D7 BPS brane on top of an $O7^-$ -plane.

B.2 T-dual relations

Looking at tables 1, 2 and 4 we find some curious behavior of the extra branes classified by cohomology, and on the RR fields classified by K-theory which do not correspond

to the known RR charges. It seems they do not obey T-duality rules. However we will see that actually they obey them. Consider a $D(q+n)$ -brane wrapped on an n -cycle of \mathbb{RP}^{8-p} , for $q \leq p$ and $p+n \leq 8-p$. The brane has position coordinates¹⁴

$$D(q+n) : \quad 0, 1, 2, \dots, q, p+1, \dots, p+n.$$

We say that a q -brane is obtained by wrapping such a brane on an n -cycle, (the suitable fraction of the D-brane is spherical in covering space), i.e, in the $p+1, \dots, p+n$ coordinates.

Now we can take T-duality on one of the coordinates defining the orientifold in two ways. Let Op be the orientifold plane along coordinates,

$$Op : \quad 0, 1, \dots, q, q+1, \dots, p$$

and T-duality is taken on the r -coordinate, with $q < r < p$. Now we have

$$\begin{aligned} O(p-1) & : \quad 0, 1, \dots, q, q+1, \dots, r-1, r+1, \dots, p \\ D(q+1+n) & : \quad 0, 1, \dots, q, r, p+1, \dots, p+n. \end{aligned} \tag{B.1}$$

If $r < q, p$, then

$$\begin{aligned} O(p-1) & : \quad 0, 1, \dots, r-1, r+1, \dots, q, q+1, \dots, p \\ D(q-1+n) & : \quad 0, 1, \dots, r-1, r+1, q, p+1, \dots, p+n \end{aligned} \tag{B.2}$$

and it corresponds to a $(q-1)$ -brane on top of an $O(p-1)$ -plane when it is wrapped on an n -cycle of $\mathbb{RP}^{8-(p-1)}$. If we denote a $D(q+n)$ -brane wrapped on a n -cycle as $D(q+n)_n$ -brane (that actually is a q -brane), then we saw that taken T-duality on some longitudinal coordinate of the orientifold plane,

$$\begin{aligned} Op & \rightarrow O(p-1) \\ Dq_n & \rightarrow \begin{cases} D(q+1)_{n+1} \\ D(q-1)_n \end{cases}, \end{aligned} \tag{B.3}$$

depending of where T-duality is taken and with q being the dimension of the D-brane in Type II theory. If $p < r < p+n$,

$$\begin{aligned} O(p+1) & : \quad 0, 1, \dots, p, r \\ D(q+n-1) & : \quad 0, 1, \dots, q, p+1, \dots, r-1, r+1, \dots, p+n \end{aligned} \tag{B.4}$$

When this brane is wrapped on an $(n-1)$ -cycle of $\mathbb{RP}^{8-(p+1)}$ it gives a q -brane. Note again that the cycle corresponds to the transverse coordinates to the orientifold plane.

¹⁴We are not considering all possible permutations of $\sigma\{0123, \dots, q\} \in \{0123 \dots p\}$, but they give the same results.

The last case is when $p < n < r$. Hence,

$$\begin{aligned} O(p+1) & : 0, 1, \dots, p, r \\ D(q+n+1) & : 0, 1, \dots, q, p+1, \dots, p+n, r \end{aligned} \quad (\text{B.5})$$

It is obtained a q -brane on top of an $O(p+1)$ -plane by wrapping this $D(q+n+1)$ -brane on an n -cycle of $\mathbb{RP}^{8-(p+1)}$. This is summarized as follows,

$$\begin{aligned} Op & \rightarrow O(p+1) \\ Dq_n & \rightarrow \begin{cases} D(q-1)_{n-1} \\ D(q+1)_n \end{cases}, \end{aligned} \quad (\text{B.6})$$

In order to illustrate the ideas, let us describe some examples. Take, for instance, the $\widehat{D1}$ -brane on top of an $O6^-$ -plane¹⁵. From table 4 we see that this brane is built by a $D2$ -brane wrapping on a 1-cycle of \mathbb{RP}^2 , or according to our notation, a $D2_1$ -brane. The array is

$$\begin{aligned} O6^- & : 0123456 \\ D2 & : 017 \end{aligned} \quad (\text{B.7})$$

After taking T-duality on some longitudinal coordinate to the orientifold (excepting the coordinate 1), the brane corresponds to a $D3$ -brane wrapping a 2-cycle of \mathbb{RP}^3 , or a $D3_2$ -brane on an $O5^-$ -plane. Again, according to table 4, this gives a $\widehat{1}$ -brane. Then by using T-duality

$$\widehat{1} \text{ (on six dimensions)} \longleftrightarrow \widehat{1} \text{ (on five dimensions)}.$$

But if T-duality is taken on coordinate 1 then

$$\begin{aligned} O5^- & : 023456 \\ D1 & : 07. \end{aligned} \quad (\text{B.8})$$

This is a $D1$ -brane wrapped on 1-cycle of \mathbb{RP}^3 , or a $D1_1$ -brane. According to our previous results this is a $\widehat{0}$ -brane. Of course we need K-theory to know to which objects are these branes related to Op^\pm -planes.

We conclude that,

- for the well known D-branes (those classified by K-theory), the relevant T-dual connecting this kind of branes, is:

$$\begin{aligned} Op & \rightarrow O(p-1) : Dq_n \rightarrow D(q-1)_n, \\ Op & \rightarrow O(p+1) : Dq_n \rightarrow D(q+1)_n. \end{aligned} \quad (\text{B.9})$$

¹⁵Again, we are using our knowledge of K-theory classification.

- The ‘extra’ branes are related each other by the following T-dual operation:

$$\begin{aligned} Op &\rightarrow O(p-1) & : D_{q_n} &\rightarrow D(q+1)_{n+1} , \\ Op &\rightarrow O(p+1) & : D_{q_n} &\rightarrow D(q-1)_{n-1} . \end{aligned} \tag{B.10}$$

At the cohomological level, the RR charge seems to be not conserved, but remember that T-duality acts over -roughly speaking- K-theory states. Looking at the tables 1 and 2, we see that for those fluxes not related to any source, T duality preserve the dimension of the region where they are turned on. Thus, we conclude that for those fields, T-duality acts as (B.10). Certainly, it is required a more exhaustive study about T-duality on RR fields in order to understand this behavior. We hope this remark could be useful in the road to elucidate this relation.

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